Grade 8

MATHEMATICS CONTENT BOOKLET: TARGETED SUPPORT

Term 4

A MESSAGE FROM THE NECT

NATIONAL EDUCATION COLLABORATION TRUST (NECT)

Dear Teachers,
This learning programme and training is provided by the National Education Collaboration Trust (NECT) on behalf of the Department of Basic Education (DBE)! We hope that this programme provides you with additional skills, methodologies and content knowledge that you can use to teach your learners more effectively.

What is NECT?
In 2012 our government launched the National Development Plan (NDP) as a way to eliminate poverty and reduce inequality by the year 2030. Improving education is an important goal in the NDP which states that 90% of learners will pass Maths, Science and languages with at least 50% by 2030. This is a very ambitious goal for the DBE to achieve on its own, so the NECT was established in 2015 to assist in improving education and to help the DBE reach the NDP goals.

The NECT has successfully brought together groups of relevant people so that we can work collaboratively to improve education. These groups include the teacher unions, businesses, religious groups, trusts, foundations and NGOs.

What are the Learning programmes?
One of the programmes that the NECT implements on behalf of the DBE is the 'District Development Programme'. This programme works directly with district officials, principals, teachers, parents and learners; you are all part of this programme!

The programme began in 2015 with a small group of schools called the Fresh Start Schools (FSS). Curriculum learning programmes were developed for Maths, Science and Language teachers in FSS who received training and support on their implementation. The FSS teachers remain part of the programme, and we encourage them to mentor and share their experience with other teachers. The FSS helped the DBE trial the NECT learning programmes so that they could be improved and used by many more teachers. NECT has already begun this embedding process.

Everyone using the learning programmes comes from one of these groups; but you are now brought together in the spirit of collaboration that defines the manner in which the NECT works. Teachers with more experience using the learning programmes will deepen their knowledge and understanding, while some teachers will be experiencing the learning programmes for the first time.

Let's work together constructively in the spirit of collaboration so that we can help South Africa eliminate poverty and improve education!

www.nect.org.za

Contents

INTRODUCTION: THREE PRINCIPLES OF TEACHING MATHEMATICS

PRINCIPLE 1: TEACHING MATHEMATICS DEVELOPMENTALLY

What is developmental teaching and what are the benefits of such an approach?

- The human mind develops through phases or stages which require learning in a certain way and which influence whether a child is ready to learn something or not.
- If learners are not ready to learn something, it may be due to the fact that they have not reached that level of development yet or they have missed something previously.
- The idea that children's thinking develop from concrete to abstract (Piaget, 1969), was refined (Miller & Mercer, 1993) to include a middle stage, namely the "concrete-representational-abstract" stages. This classification is a handy tool for mathematics teaching. We do not need to force all topics to follow this sequence exactly, but at the primary level it is especially valuable to establish new concepts following this order.
- This classification gives a tool in the hands of the teacher, not only to develop children's mathematical thinking, but also to fall back to a previous phase if the learner has missed something previously.
- The need for concrete experiences and the use of concrete objects in learning, may gradually pass as learners develop past the Foundation Phase. However, the representational and abstract development phases are both very much present in learning mathematics at the Intermediate and Senior Phases.

How can this approach be implemented practically?

The table on page 7 illustrates how a developmental approach to mathematics teaching may be implemented practically, with examples from several content areas.

What does this look like in the booklet?

Throughout the booklets, within the topics, suggestions are made to implement this principle in the classroom:

- Where applicable, we suggest an initial concrete way of teaching and learning a concept and we provide educational resources at the end of the lesson plan or topic to assist teachers in introducing the idea concretely.
- Where applicable, we provide pictures (representational/semi-concrete) and/or diagrams (representational/semi-abstract). It may be placed at the clarification of terminology section, within the topic itself or at the end of the topic as an educational resource.
- In all cases we provide the symbolic (abstract) way of teaching and learning the concept, since this is, developmentally speaking, where we primarily aim to be for learners to master mathematics.

PRINCIPLE 2: TEACHING MATHEMATICS MULTI-MODALLY

What is multi-modal teaching and what are the benefits of such an approach?

- We suggest that teachers present mathematics topics in three forms to provide for all learners' learning styles and preferences. They (a) explain the idea by speaking about a topic, (b) illustrate it by showing pictures or diagrams and finally (c) present the idea by symbolising it in numbers and mathematical symbols.
- Teaching in more than one form (multi-modal teaching) includes hearing the same mathematical idea in spoken words (auditory mode), seeing it in a picture or a diagram (visual mode) and writing it in a mathematical way (symbolic mode).
- Learners differ in the way they learn and understand mathematical ideas. For one learner it is easier to understand through hearing and for the other through seeing. That is why we open both pathways to the symbolic mode – because here they do not have a choice, they all have to reach that mode, be it through hearing or seeing.

How can this approach be implemented practically?

The table on page 8 illustrates how a multi-modal approach to mathematics teaching may be implemented practically, with examples from several content areas.

What does this look like in the booklet?

Throughout the booklets, within the topics at the Senior Phase, we suggest ways to apply this principle in the classroom:

- The verbal explanations under each topic and within each lesson plan, provide the "speak it" or auditory mode.
- The pictures and diagrams give suggestions for the "show it" mode (visual mode).
- The calculations, exercises and assessments under each topic and within each lesson plan, provide the "symbol it" or symbolic mode of representation.

PRINCIPLE 3: SEQUENTIAL TEACHING

What is sequential teaching and what are the benefits of such an approach?

- Learners with weak basic skills in mathematics will find future topics increasingly difficult. A solid foundation is required for a good fundamental understanding.
- In order to build a solid foundation in maths, we teach concepts systematically. If we teach concepts out of that order, it can lead to difficulties in grasping concepts.
- Systematic teaching according to CAPS builds progressive understanding and skills.
- In turn, this builds confidence in the principles of a topic before he/she is expected to apply the knowledge and proceed to a higher level.
- We have to continuously review and reinforce previously learned skills and concepts.
- If learners link new topics to previous knowledge (past), understand the reasons why they learn a topic (present) and know how they will use the knowledge in their lives ahead (future), it can help to motivate them and to remove many barriers to learning.

How can this approach be implemented practically?

If a few learners in your class are not grasping a concept, you as the teacher need to take them aside and teach them the concept again (perhaps at a break or after school).

If the entire class are battling with a concept, it will need to be taught again, however this could cause difficulties in trying to keep on track and complete the curriculum in time.

To finish the year's work within the required time and to also revise topics, we suggest:

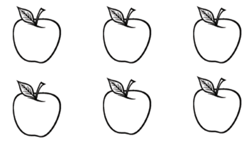
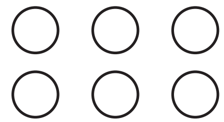
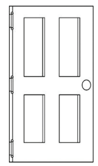
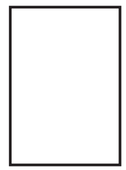
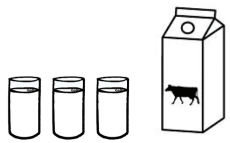
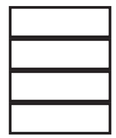
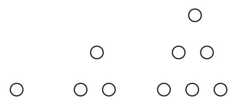
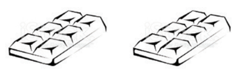
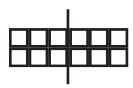
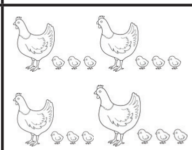
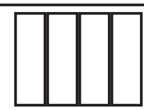
- Using some of the time of topics with a more generous time allocation, to assist learners to form a deeper understanding of a concept, but also to catch up on time missed due to remediating and re-teaching of a previous topic.
- Giving out revision work to learners a week or two prior to the start of a new topic. For example, in Grade 8, before you are teaching Data Handling, you give learners a homework worksheet on basic skills from data handling as covered in Grade 7, to revise the skills that are required for the Grade 8 approach to the topic.

What does this look like in the booklet?

At the beginning of each topic, there are two parts that detail

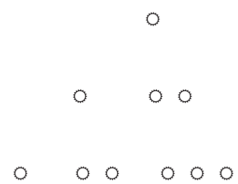
- The SEQUENTIAL TEACHING TABLE lays out the knowledge and skills covered in the previous grade, in the current grade and in the next grade.
- The LOOKING BACK and LOOKING FORWARD summarises the relevant knowledge and skills that were covered in the previous grade or phase and that will be developed in the next grade or phase.

THREE-STEP APPROACH TO MATHEMATICS TEACHING: CONCRETE-REPRESENTATIONAL-ABSTRACT

CONCRETE: IT IS THE REAL THING		REPRESENTATIONAL: IT LOOKS LIKE THE REAL THING		ABSTRACT: IT IS A SYMBOL FOR THE REAL THING	
Mathematical topic	Real or physical For example:	Picture	Diagram	Number (with or without unit)	Calculation or operation, general form, rule, formulae
Counting	Physical objects like apples that can be held and moved			6 apples	$2 \times 3 = 6$ or $2+2+2=6$ or $\frac{1}{2}$ of $6 = 3$ or $\frac{2}{3}$ of $6 = 4$
Length or distance	The door of the classroom that can be measured physically			80 cm wide 195 cm high	Perimeter: $2L + 2W = 390 + 160$ $= 550cm$ Area: $L \times W = 195 \times 80$ $= 15\ 600cm^2$ $= 1.56m^2$
Capacity	A box with milk that can be poured into glasses			1 litre box 250 ml glass	$4 \times 250ml = 1\ 000ml$ $= 1$ litre or 1 litre $\div 4 = 0.25$ litre
Patterns	Building blocks			1: 3: 6...	$\frac{n(n+1)}{2}$
Fraction	Chocolate bar			$\frac{6}{12}$	$\frac{6}{12} = \frac{1}{2}$ or $\frac{1}{2}$ of $12 = 6$
Ratio	Hens and chickens		* *** * *** * *** * ***	4:12	$4 : 12 = 1 : 3$ Of 52 fowls ¼ are hens and ¾ are chickens. ie 13 hens. 39 chickens
Mass	A block of margarine			500g	$500g = 0{,}5\ kg$ or calculations like 2 ½ blocks = 1.25kg

Teaching progresses from concrete -> to -> abstract. In case of problems, we fall back <- to diagrams, pictures, physically.

MODES OF PRESENTING MATHEMATICS WHEN WE TEACH AND BUILD UP NEW CONCEPTS

Examples	**SPEAK IT - explain**	**SHOW IT - embody**	**SYMBOL IT - enable**
	• To introduce terminology • To support auditory learning • To link mathematics to real life	• To help storing and retrieving ideas • To help visual learning • To condense information to one image	• To promote mathematical thinking • To convert situations to mathematics • To enable calculations
IP: Geometric patterns	"If shapes grow or shrink in the same way each time, it forms a geometric pattern or sequence. We can find the rule of change and describe it in words. If there is a property in the shapes that we can count, each term of the sequence has a number value" "You will be asked to draw the next term of the pattern, or to say how a certain term of the pattern would look. You may also be given a number value and you may be asked, which term of the pattern has this value?"	 T1 T2 T3 T4 Question: (a) Draw the next term in this pattern. (b) Describe this pattern. What is the value of the 9th term of this pattern (T9)? **Which term has a value of 120?** To draw up to the 9th term of the sequence and to find out which term has a value of 120, is slow. One is now almost forced to deal with this problem in a symbolic way.	Say out loud: 1; 3; 6… 1; 3; 6; 10… 1; 3; 6;10;15 Inspect the number values of terms: T1: 1 = 1 T2: 3 = 1+2 T3: 6 = 1+2+3 T4: 10 = 1+2+3+4 T9: 45 = 1+2+3+4+5+6+7+8+9 General rule: The value of term n is the sum of n number of consecutive numbers, starting at 1.
SP: Grouping the terms of an algebraic expression	"We can simplify an algebraic expression by grouping like terms together. We therefore have to know how to spot like terms. Let us say we have to sort fruit in a number of baskets and explain the variables or the unknowns in terms of fruits. Try to imagine the following pictures in your mind:"	Although not in a real picture, we can paint a mind picture to help us understand the principle of classification: • Basket with green apples (a) • Basket with green pears (b) • Basket with green apples and green pears (ab) • Basket with yellow apples (a^2) • Basket with yellow apples and green pears (a^2b) Or in diagram form a □ b ○ ab □○ a^2 ■ a^2b ■○	Group and simplify the following expression: $4b - a^2 + 3a^2b - 2ab - 3a + 4b + 5a - a - 2ab + 2a^2b + a^2b$ $= -3a + 5a - a + 4b + 4b - 2ab - 2ab - a^2 + 3a^2b + 2a^2b + a^2b$ $= a + 8b - 4ab - a^2 + 6a^2b$

TOPIC 1: FUNCTIONS AND RELATIONSHIPS

INTRODUCTION

- This unit runs for 6 hours.
- It is part of the content area, ‘Patterns, Functions and Algebra’ and counts for 30% in the final exam.
- This topic was started in Term 1 so the learners should already have a good understanding of the basics involved and be ready to move on to the next level covered in Term 4. Mastering the idea of the connections between functions and their different formats is a basic necessity to having a better understanding of functions for future years.
- It is important to note that this section is linked closely to Algebra and Equations. Now that both of these topics have been done in previous terms, learners should find this section somewhat easier.
- Learners need to come to an understanding that flow diagrams, tables, formulae and equations can all be used to represent PATTERNS.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 7	GRADE 8	GRADE 9/ FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Determine input values, output values and rules for patterns and relationships using flow diagrams, tables and formulae • Determine, interpret and justify equivalence of different descriptions of the same relationship or rule presented verbally, in flow diagrams, in tables, by formulae and by number sentences	• Determine input values, output values and rules for patterns and relationships using flow diagrams, tables, formulae and equations • Determine, interpret and justify equivalence of different descriptions of the same relationship or rule presented verbally, in flow diagrams, in tables, by formulae and by equations	• Determine input values, output values and rules for patterns and relationships using flow diagrams, tables, formulae and equations • Determine, interpret and justify equivalence of different descriptions of the same relationship or rule presented verbally, in flow diagrams, in tables, by formulae, by equations and by graphs on a Cartesian Plane

GLOSSARY OF TERMS

Term	Explanation / Diagram
Function	A mathematical condition or rule linking the input to the output.
Input	The number/value that was chosen to replace the variable in an expression.
Output	The output is dependent on the input – it is the answer once the operation has been performed according to the expression given.
Equation	A mathematical sentence built from an algebraic expression using an equal sign. For example: $2a + 3 = 10$
Expression	A mathematical model which represents a situation. It can include variables (letters), constants and operations. For example: $2b + 3c$
Flow Diagram	A diagram representing a sequence of movements to be performed on a given value.
Algebraic Rule	An expression representing a rule to be performed on the variable. For example: $3m + 1$ (Multiply the number represented by 'm' by 3 then add 1 to the answer)
Inverse Operation	The opposite operation that will 'undo' an operation that has been performed. Addition and subtraction are the inverse operation of each other. Multiplication and Division are the inverse operation of each other.

SUMMARY OF KEY CONCEPTS

A function is a special rule or relationship between values

Inputs and Outputs

Every value you put into a function (input) has a specific value that comes out (output) after one or more operations have been performed.

Flow Diagrams

Flow diagrams show how input numbers are changed to become output numbers.

Mathematical rules are used to show what operations have been applied to the input numbers in order to get the output numbers.
A flow diagram is similar to an equation, just written differently.

For example:

Equation	Flow Diagram
$5 + 2 = 7$	$5 \rightarrow +2 \rightarrow =7$

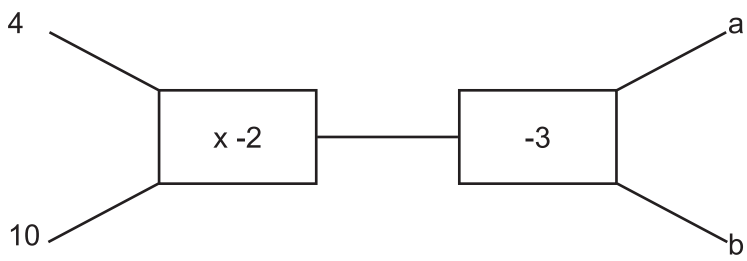
Flow diagrams can be changed into equations when solving is required.

Flow Diagram	Equation
$x \rightarrow +2 \rightarrow =7$	$x + 2 = 7$

Patterns can be represented algebraically. This means that variables (letters that can represent many values) will be used.

Finding output when given the input and the rule

We can use flow diagrams to find missing values.
For example:



To find 'a': Start with 4, multiply by -2 and then subtract 3 (a = -11)
To find 'b': Start with 10, multiply by -2 and then subtract 3 (b = -23)

Point out to learners that their knowledge of substitution in algebra is required.

NOTE: A flow diagram is not the only way of representing functions and relationships. Tables, formulae and equations can also be used. In the following examples, also take note that the domain (input value) is also stated. This can include natural numbers, whole numbers, integers or rational numbers.

This will give learners an opportunity to revise and build their awareness of the number system. It also provides an opportunity to revise working with both positive and negative values as well as fractions again.

For example:
Find the output (b) using the given formula. The input (a) is an element of rational numbers (a ∈ Q)

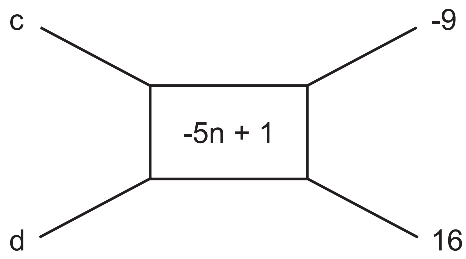
a	-2	$0{,}5$	$\frac{3}{4}$
$b = 4a - 1$			

$b = 4a - 1$ $b = 4(-2) - 1$ $b = -8 - 1$ $b = -9$	$b = 4a - 1$ $b = 4(0{,}5) - 1$ $b = 2 - 1$ $b = 1$	$b = 4a - 1$ $b = 4(\frac{3}{4}) - 1$ $b = 3 - 1$ $b = 2$

Finding input when given the output and the rule

Since we are working backwards, we need to work with INVERSE operations in order to 'undo' the expression.

For example:



First consider the meaning of -5n + 1: Multiply by -5, then add 1

To find 'c': Using inverse operations:
Start with -9, subtract 1 (inverse operation to addition)
then divide by -5 (inverse operation to multiplication) (c = 2)

To find 'd': using inverse operations:
Start with 16, subtract 1
then divide by -5 (d = -3)

Point out to learners that their knowledge of solving equations and working with inverse operations is required.

Teaching Tip:
Encourage learners to check their answers by working forwards again.

A further example of finding input using a formula:

Using the formula, Area of $\Delta = \frac{1}{2}$ base $\times \perp$height , find the base of a triangle if the area is $15cm^2$ and the height is 3cm.

Solution:

Area of $\Delta = \frac{1}{2}$ base $\times \perp$height

$15cm^2 = \frac{1}{2}$ base ×3cm — Note that this requires solving an equation. The base is the unknown and we need to get it on its own.

$5cm = \frac{1}{2}$ base — Both sides were divided by 3cm

10cm = base — Both sides were multiplied by 2 to 'remove/undo' the $\frac{1}{2}$

Finding the rule when given the input and output

To find the rule, the input and output numbers both need to be considered.

Point out to learners that a function is essentially a number pattern which was covered in Term 1. A recap of patterns (and in particular linear patterns) would be a good idea at this stage.

There are two types of rules to expect:

1. To get from the input to the output only one operation has been performed.

   For example:

a	3	5	7	9
b	6	10	14	18

   To get from the input (a) to the output (b), we need to multiply by 2
   The rule is: a x 2 = b   (This is better written as 2a = b)

   A further example:

p	-1	0	1	2
q	4	5	6	7

   To get from the input (p) to the output (q), we need to add 5
   The rule is: p + 5 = q

2. To get from the input to the output two operations have been performed.

   For example:

f	1	2	3	4
g	4	7	10	13

Steps to follow to find rule:

- Look at the output numbers and find the common difference (in this case 3) 7 - 4 = 3 and 10 - 7 = 3 etc
- This tells us that the first part of the rule is 'times 3'
- Look at the input numbers and times them by 3. What still needs to be done to get to the output? (in this case add 1)
- Therefore, the rule is: 'times 3, add 1',
- Rule: f x 3 + 1 = g, which is better written as: 3f + 1 = g

A further example:

x	1	2	3	4
y	1	6	11	16

- The common difference between the output values is ‘5’
- This tells us that the first part of the rule is ‘times 5’
- Times the first input number by 5 (1 x 5) and check what needs to be done to get the output (we need to subtract 4). Confirm this by checking if the same thing occurs with the next input (2 x 5 = 10. To get 6 we will need to subtract 4)
- Therefore the rule is: x 5 then subtract 4 which is better written as $y = 5x - 4$

Equivalent forms

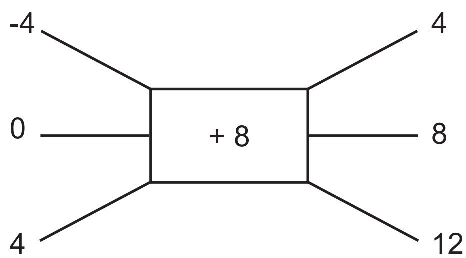
All functions can be represented either:
- In words
- In table form
- As a flow diagram
- Using equations or formulae

Learners need to recognise all of these forms and be able to change between one or another.

This is an example using one function (rule):
In words: y is the sum of 8 and x
In table form:

x	-4	0	4
y	4	8	12

As a flow diagram:



As an equation: $y = x + 8$

TOPIC 2: ALGEBRAIC EQUATIONS

INTRODUCTION

- This unit runs for 3 hours.
- It is part of the content area, 'Patterns, Functions and Algebra' and counts for 30% in the final exam.
- The unit covers a revision of the equations covered in the previous two terms.
- It is important to note that in this term the focus is on using substitution in equations to generate tables of ordered pairs. This is an important skill for both algebra and graphs.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 7	GRADE 8	GRADE 9/ FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Write number sentences to describe problem situations • Analyse and interpret number sentences • Solve and complete number sentences by inspection and trial and error • Determine the numerical value of an expression by substitution • Identify variables and constants in given formulae or equations	• Set up equations to describe problem situations • Analyse and interpret equations that describe a situation • Solve equations by inspection • Determine the numerical value of an expression by substitution • Identify variables and constants in given formulae or equations • Use substitution in equations to generate ordered pairs • Extend solving equations to include using additive inverses, multiplicative inverses and the laws of exponents	• Set up equations to describe problem situations • Analyse and interpret equations • Solve equations by inspection • Solve equations using additive inverses, multiplicative inverses and the laws of exponents • Determine the numerical value of an expression by substitution • Use substitution in equations to generate ordered pairs • Solve equations using factorisation • Solve equations in the form: a product of factors = 0

GLOSSARY OF TERMS

Term	Explanation / Diagram
Equation	A mathematical statement with an equal sign that includes a variable. For example: $3x - 5 = 20$
Expression	An algebraic statement consisting of terms with variables and constants. There is no equal sign. For example: $3a + 2b$
Formula	A formula is used to calculate a specific type of answer and has variables that represent a certain kind of value. For example: Area = . l x b This formula finds area of a rectangle and only measurements can replace the l and the b.
Variable	Letters of the alphabet which could represent different values. For example: In the expression $m + 2$. m is a variable and could be replaced by a number in order to calculate the answer when m is equal to that specific number. Variables can change values.
Like Terms	Terms that have exactly the same variables. For example: 2a and 4a are like terms and can be added or subtracted 3abc and 10abc are like terms and can be added or subtracted
Inverse Operation	The opposite operation that will 'undo' an operation that has been performed. Addition and subtraction are the inverse operation of each other. Multiplication and Division are the inverse operation of each other.
Substitution	Replacing the variable with a number. Used in Equations to check if the answer is correct by checking if the Left Hand Side (LHS) is equal to the Right Hand Side (RHS) of the original equation.

SUMMARY OF KEY CONCEPTS

It is important that some time be spent on revising equations from Term 1 and Term 2.
This should include:

- Solving equations using additive and multiplicative inverses
- Solving equations by using the laws of exponents
- Setting up equations to describe given situations
- Analysing and interpreting equations

Notes on all of the above can be found in Term 1 and 2's Content Booklet.

A reminder of the steps to be followed to solve equations:

- Use inverse operations to get the unknown terms on one side (usually the left hand side)
- Use inverse operations to get the constant terms on one side (usually the right hand side)
- Simplify each side of the equation by collecting like terms if possible
- Use inverse operations to get the unknown variable by itself and hence solve for the unknown in the given equation

Detailed examples including full steps and explanations can be found in both Term 1 and Term 2's booklets.

A worked example:

$$
\begin{aligned}
3x + 4 &= -2 - x \\
3x + x &= -2 - 4 \\
4x &= -6 \\
\frac{4x}{4} &= \frac{-6}{4} \\
x &= \frac{-3}{2}
\end{aligned}
$$

Solving equations by using the laws of exponents

Consider the following statement:

$5^6 = 5^x$

It should be easy to see that $x = 6$. This is because the bases are the same.
If you ever need to solve for a variable in the exponent position, it is always necessary to get the bases to be equal on each side.

Examples:

1. $4^x = 64$
   $4^x = 4^3$
   $\therefore\ x = 3$

2. $2.2^x = 16$ change all bases to 2
   $2.2^x = 2^4$ use rules of exponents to simplify
   $2^{1+x} = 2^4$ once bases are the same, make the exponents equal and solve
   $\therefore 1 + x = 4$
   $x = 3$

NOTE: The second example is quite difficult and more focus should be on questions similar to the first example.

Generating tables of ordered pairs

This skill will be particularly useful in the next topic for the term. Ordered pairs will need to be generated from equations in order to plot points on a Cartesian plane.

Learners need to be able to substitute given values (input) to find the matching values (output) for the function given.

For example: Given the equation $y = 3x + 2$ complete the following table:

x	-2	-1	0	1	2
y					

$y = 3x + 2$ $y = 3(-2) + 2$ $y = -6 + 2$ $y = -4$	$y = 3x + 2$ $y = 3(-1) + 2$ $y = -3 + 2$ $y = -1$	$y = 3x + 2$ $y = 3(0) + 2$ $y = 0 + 2$ $y = 2$	$y = 3x + 2$ $y = 3(1) + 2$ $y = 3 + 2$ $y = 5$	$y = 3x + 2$ $y = 3(2) + 2$ $y = 6 + 2$ $y = 8$

The table can now be completed:

x	-2	-1	0	1	2
y	-4	-1	2	5	8

Learners also need to be able to find the input value when given the output value.

For example:
Given the equation $y = -2x + 1$ complete the following table:

x	-3	0	1		
y				-3	5

The first three calculations are the same as the previous example (using substitution).

$y = -2\,(-3) + 1 = 7$

$y = -2\,(0) + 1 = 1$

$y = -2\,(1) + 1 = -1$

In order to find the final two solutions, we need to solve an equation.

$\begin{aligned} y &= -2x+1 \\ -3 &= -2x+1 \\ 2x &= 1+3 \\ 2x &= 4 \\ x &= 2 \end{aligned}$	$\begin{aligned} y &= -2x+1 \\ 5 &= -2x+1 \\ 2x &= 1-5 \\ 2x &= -4 \\ x &= -2 \end{aligned}$

The table can now be completed:

x	-3	0	1	2	-2
y	7	1	-1	-3	5

TOPIC 3: GRAPHS

INTRODUCTION

- This unit runs for 9 hours.
- It is part of the content area, 'Patterns, Functions and Algebra' and counts for 30% in the final exam.
- The unit covers the interpretation of graphs and the drawing of graphs.
- It is important to note that this section can be linked to the Data Handling section as well as Functions and Relationships. This should be pointed out to the learners where possible.
- Graphs are visual representations of numerical systems and equations.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/ GRADE 7	GRADE 8	GRADE 9/ FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Analyse and interpret global graphs of problem situations with a focus on linear and non-linear and the terms constant, increasing and decreasing • Draw global graphs from given descriptions of a problem situation, identifying the features listed above	• Analyse and interpret global graphs of problem situations with a focus on linear and non-linear and the terms constant, increasing and decreasing • Focus on the features of a graph including maximum, minimum, discrete and continuous • Draw global graphs from given descriptions of a problem situation, identifying the features listed above • Use tables or ordered pairs to plot points and draw graphs on the Cartesian plane	• Analyse and interpret global graphs of problem situations with a focus on linear and non-linear and the terms constant, increasing and decreasing; maximum, minimum, discrete and continuous • Extend the above with a special focus on the following features of a linear graph: intercepts and gradient • Draw global graphs from given descriptions of a problem situation, identifying the features listed above • Use tables or ordered pairs to plot points and draw graphs on the Cartesian plane • Draw linear graphs from given equations • Determine equations of linear graphs

GLOSSARY OF TERMS

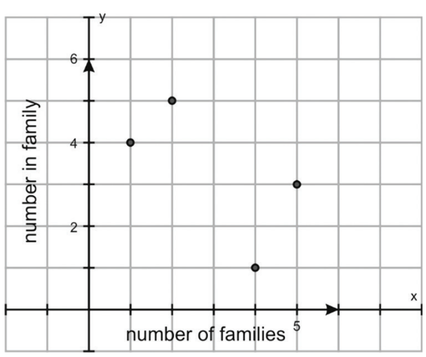
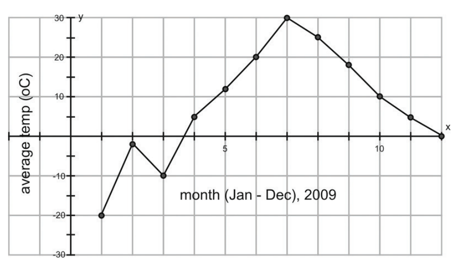
Term	Explanation / Diagram
Discrete	Distinct and separate data. Discrete data is counted. Example: Number of books in a classroom
Continuous	Data that can take on any value. Continuous data is measured. For example: The height of a person
Cartesian Plane	Two lines – one horizontal (x-axis) and one vertical (y-axis) Both axes are labelled with positive and negative numbers The two axes cross each other to form four areas called quadrants.
Axes	These are straight lines joined at the origin, these are used to determine the exact points required to draw functions.
Origin	The intersection of the axes (0;0) is the coordinate that represents the origin.
Co-ordinate	This is a unique ordered pair of numbers that identifies a point on the Cartesian plane. The first number in the ordered pair identifies the position with regard to the x-axis and then the second identifies the relation to the y-axis.
Linear	A graph that follows the general form of a line
Non-linear	A graph that does not follow the general form of a line (There are many other descriptions which will be explored in later grades)
Increasing	A graph is increasing if it is going uphill from left to right A graph can increase then change to become a decreasing graph
Decreasing	A graph is decreasing if it is going downhill from left to right A graph can decrease then change to become an increasing graph
Maximum	The highest possible point of a graph (function) It is directly related to the y-value
Minimum	The lowest possible point of a graph (function) It is directly related to the y-value

SUMMARY OF KEY CONCEPTS

Interpreting graphs

Being able to interpret graphs representing real life situations and being able to draw graphs are important skills to have.

Both discrete data and continuous data can be represented in graphs.

	Discrete data	Continuous data
Definition	A set of data is discrete if the values in the set are distinct and separate	A set of data is continuous if the values can take on any value within a certain range
Examples	The number of people in your class The number of TV's in a home The number of questions in a test	The height of horses in a race Time taken to run a race Temperature in a given city
Useful to know	Discrete data is counted	Continuous data usually requires a measuring device
Graph descriptions	The points plotted are separate Only the points themselves have meaning (ie we can't read information from the spaces between the points) Points are not connected	The points plotted are connected with a continuous line Every point has meaning (ie the points that have been plotted from the data as well as any part of the joined line) It is possible to draw a continuous graph without lifting your pencil
Graphs		

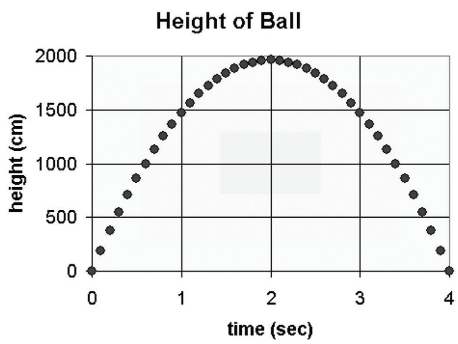
Maximum and Minimum

Some graphs have a maximum or a minimum value.

Teaching Tip: To assist learners in finding the maximum or minimum point of a graph, ask them to lay their ruler down below the graph and slowly move it upwards. The first time their ruler touches the graph they will be finding the minimum point (which will be read from the y-axis). The last time their ruler touches the graph they will be finding the maximum point (which will be read from the y-axis).

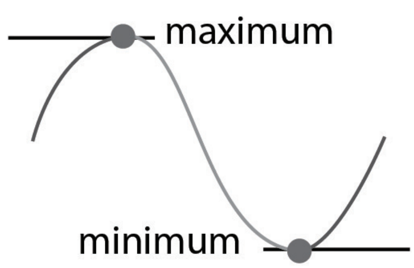
NOTE: Not all graphs have these points.
For example, a straight line graph (where x is an element of real numbers) has neither.

The following graph represents a ball being bounced:

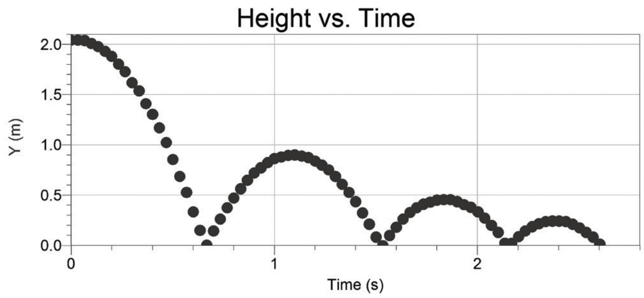


Note that it has a MAXIMUM point which represents the highest point the ball reached (in this case 2000cm at the 2 second mark)

The following diagram shows a graph that has both a maximum AND a minimum point.



The following graph represents a situation of a ball bouncing:

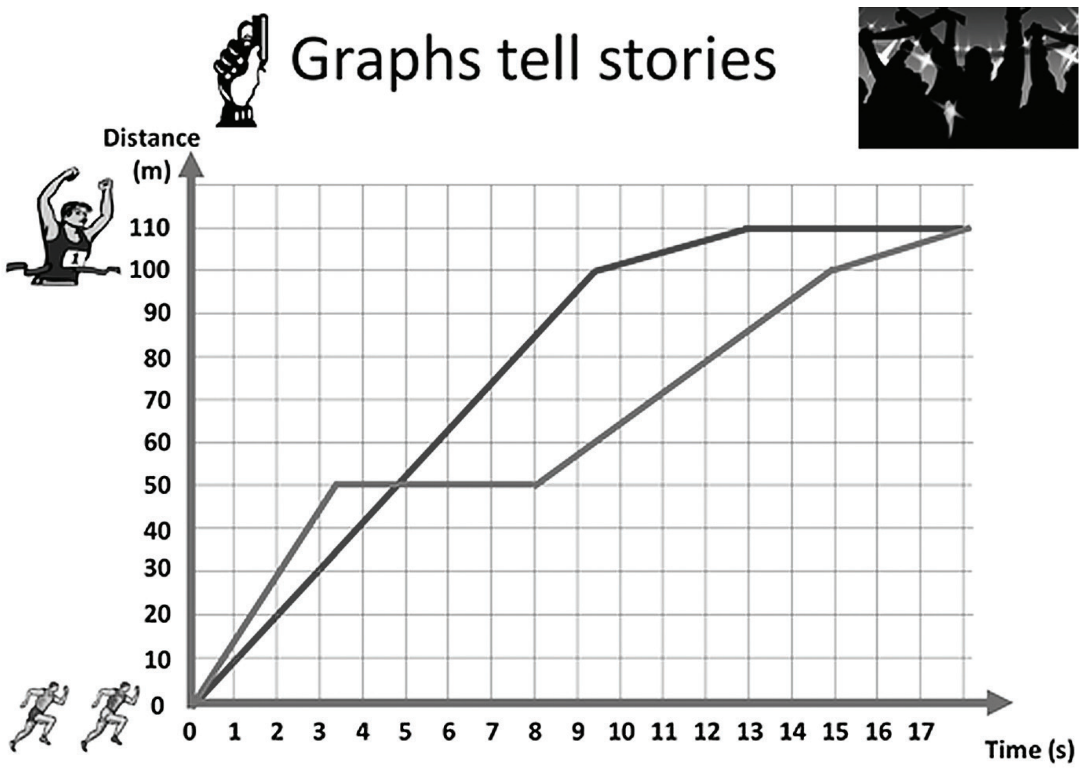


Note that there are a few minimum points in the graph and these all represent the ball hitting the ground each time that it bounces. The ground is the lowest possible point that the ball can go. The maximum point on is graph is at 2m – the height the ball is first dropped from. No matter how many times the ball bounces, it will never reach the height of 2m again.

The graph below representing some unemployment statistics over a period of 35 years also has a maximum and a minimum point. The minimum point is approximately 1,7% in 1971 and the maximum point is approximately 11% in about 1993.

Drawing graphs

(i) Global Graphs



This graph tells the story of two runners.
Learners need to be able to understand (and tell a story) from a graph such as this one and also be able to take information representing a situation and put it into a visual representation.
Some information which should be noted from this graph:

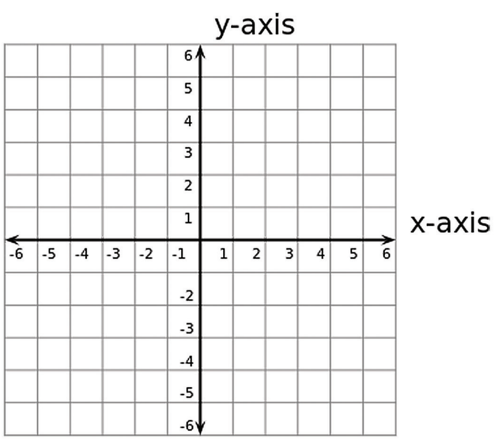
- Both runners start at the same time (0;0) – they have not yet travelled any distance (0m) and no time has yet elapsed (0s)
- One runner starts out faster than the other runner. His line is above the slower runner. At 3 seconds, the faster runner (who will be called Runner A from now on) has travelled approximately 45m and the slower runner (who will be called Runner B from now on) has travelled 30m.
- From just past 3 seconds until 8 seconds Runner A does not travel any further than the 50m mark. He seems to have stopped.
- Just before the 5 second mark, Runner B overtakes Runner A.
- Runner A runs again 8 seconds into the race.
- At 13 seconds, Runner B reaches 110m and stops.
- 15 seconds into the race, Runner A has reached 100m and 3 seconds later (18 seconds from the start) he reaches 110m.

When information is given and a graph needs to be drawn, ensure the following points are considered:

- Draw a neat set of axes and label each one
- Decide what scale needs to be used on each axes and fill in the numbers required.
- Give the graph a heading
- Mark off the important points
- If the data is continuous, join the points. If the data is discrete, do not join the points
- Always ask, 'would someone understand all the information represented and be able to tell a story from the graph without hearing an explanation first'

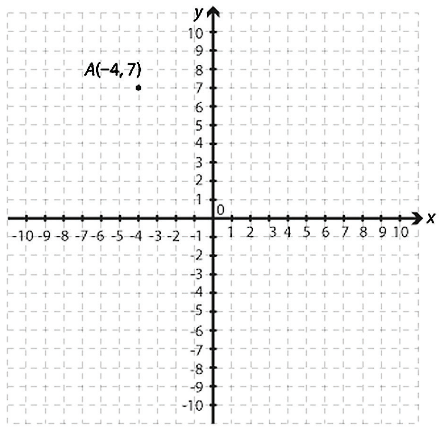
(ii) Algebraic graphs on a Cartesian plane

- All algebraic functions (graphs) are drawn on a Cartesian Plane
- The plane is made up of two axes: the x-axis and the y-axis
- The x-axis is the horizontal axis and the y-axis is the vertical axis
- The two axes meet at the origin which is the point (0 ; 0)
- Plotting points is an important skill for drawing graphs. All points are made up of an x-value and a y-value. These points are also known as ordered pairs.



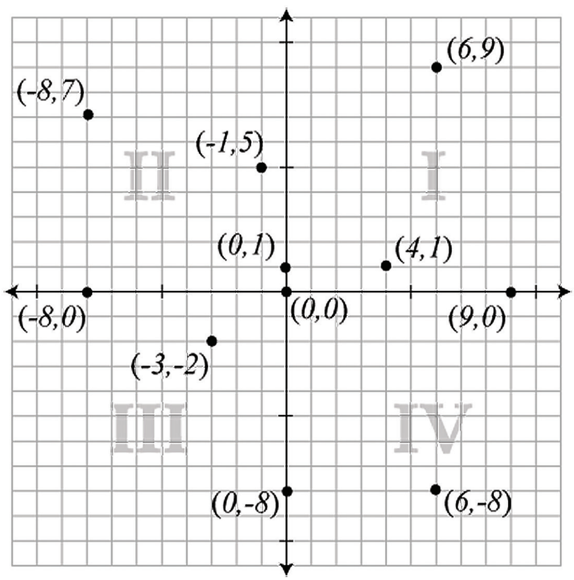
Each co-ordinate is made up of an x co-ordinate and a y co-ordinate. For example (-4 ;7)

To plot this point, find -4 on the x-axis and 7 on the y-axis then look where the points would meet. Another way of looking at it is to start at the origin, move 4 units left (negative) and 7 units up (positive).
Note how the point has been plotted and labelled.



Note carefully the following plotted points.

Learners need to practice plotting as many points as possible. Until this skill is mastered and done with ease, moving on to drawing graphs should not be done



Teaching Tip: For learners who may get confused with which number comes first, remind them that it is in alphabetical order. The x-value is always first and the y-value is always second.

NOTE: It is important for the learners to spend time practicing plotting points correctly. If this isn't achieved at a high level, the drawing of graphs this year and in future years will be increasingly difficult for them. A good way to practice is by plotting points that form a picture. They are easy for you as the teacher to check and the learners can also see if something doesn't look correct. There are two such tasks in the resource section at the end of the section.

Using tables to draw graphs on a Cartesian Plane

This section is directly connected to the topic covered earlier in the year on functions and relationships.

For example: Draw the graph of the function $y = 2x - 4$
Step 1: Draw up a table with a row for x values and a row for y values and fill in some values for x (usually -2, -1, 0, 1 and 2 so there is a range of negative and positive integers)

x	-2	-1	0	1	2
y					

Step 2: Using substitution, find the corresponding values of y that match with each of the values chosen for x

$y = 2x - 4$ $y = 2(-2) - 4$ $y = -4 - 4$ $y = -8$	$y = 2x - 4$ $y = 2(-1) - 4$ $y = -2 - 4$ $y = -6$	$y = 2x - 4$ $y = 2(0) - 4$ $y = 0 - 4$ $y = -4$	$y = 2x - 4$ $y = 2(1) - 4$ $y = 2 - 4$ $y = -2$	$y = 2x - 4$ $y = 2(2) - 4$ $y = 4 - 4$ $y = 0$

Tha table can now be completed:

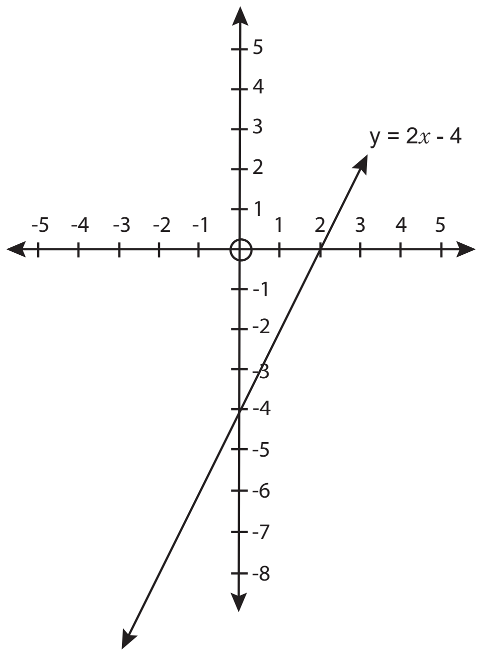
x	-2	-1	0	1	2
y	-8	-6	-4	-2	0

There are now 5 co-ordinates that can be plotted on the Cartesian plane:

(- 2 ; - 8) ; (- 1 ; - 6) ; (0 ; - 4) ; (1 ; - 2) ; (2 ; 0)

Step 3: Draw the Cartesian Plane. Plot the points. Draw the line to join the points. Always extend the line past the points (to show that other values for x could have been used – remember that you only found five of the many possible co-ordinates that could lie on this graph)

Step 4: Put arrows on the ends of the line to show that it is continuous and write the equation on the line.



A further example with a non-linear function:

Draw the graph of the function $y = x^2 - 4$

NOTE: Although learners will only be formally introduced to the quadratic function in Grade 10, it is merely used here as an exercise in the understanding that any function can be represented on a Cartesian Plane by substituting x-values, finding the corresponding y-values and hence finding a set of ordered pairs that can be plotted.

Step 1: Draw up a table with a row for x values and a row for y values and fill in some values for x (usually -2, -1, 0, 1 and 2 so there is a range of negative and positive integers)

x	-2	-1	0	1	2
y					

Step 2: Using substitution, find the corresponding values of y that match with each of the values chosen for x

$y = x^2 - 4$ $y = (-2)^2 - 4$ $y = 4 - 4$ $y = 0$	$y = x^2 - 4$ $y = (-1)^2 - 4$ $y = 1 - 4$ $y = -3$	$y = x^2 - 4$ $y = (0)^2 - 4$ $y = 0 - 4$ $y = -4$	$y = x^2 - 4$ $y = (1)^2 - 4$ $y = 1 - 4$ $y = -3$	$y = x^2 - 4$ $y = (2)^2 - 4$ $y = 4 - 4$ $y = 0$

The table can now be completed:

x	-2	-1	0	1	2
y	0	-3	-4	-3	0

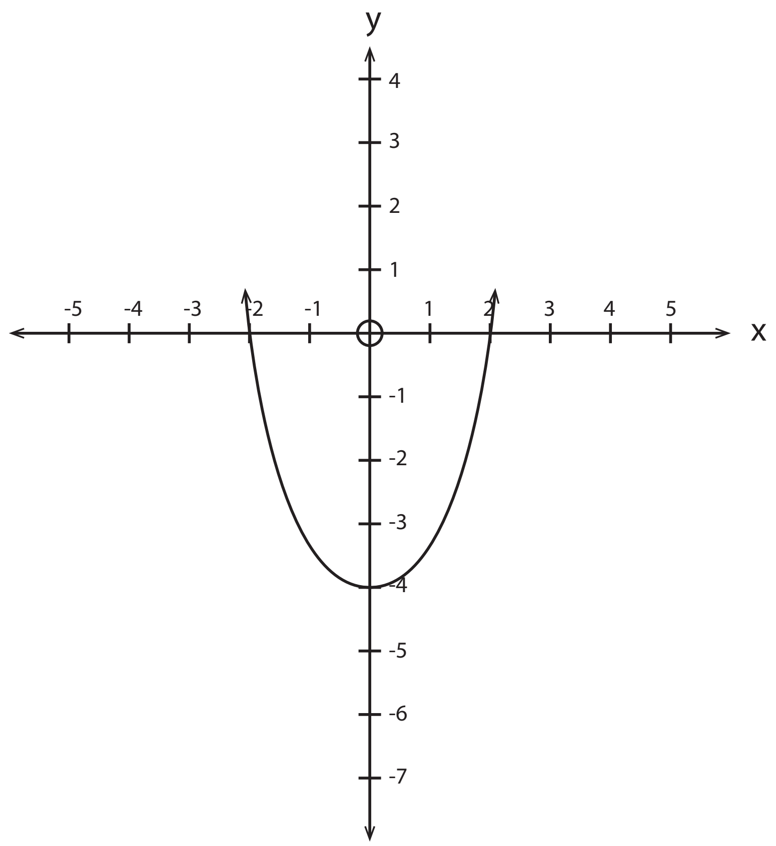
There are now 5 co-ordinates that can be plotted on the Cartesian plane:

(- 2 ; 0) ; (- 1 ; - 3) ; (0 ; - 4) ; (1 ; - 3) ; (2 ; 0)

Teaching Tip: Stress to learners that these are only 5 of an infinite number of points that have been found. This is the reason the points are joined with a solid line as this then represents all the possible points that could have been found.

Step 3: Draw the Cartesian Plane. Plot the points and join them. As this is not a linear function, the points must be drawn freehand (without a ruler). It should be a curved graph. Always extend the line past the points (to show that other values for x could have been used – remember that you only found five of the many possible co-ordinates that could lie on this graph)

Step 4: Put arrows on the ends of the line to show that it is continuous and write the equation on the function.



RESOURCES

The first worksheet uses quadrant 1 (positive numbers only) and the second one uses all four quadrants so learners can practice all types of co-ordinate plotting.

Note: There are also 2 worksheets available in the Term 4 Grade 9 Content Booklet

Name:______________________________

River Riding

NOTE: In each section, do NOT connect the last point back to first point.

(X, Y)

- ☐ (0, 1)
- ☐ (2, 3)
- ☐ (4, 3)
- ☐ (3, 2)
- ☐ (4, 1)

STOP

- ☐ (24, 17)
- ☐ (24, 18)
- ☐ (23, 19)
- ☐ (22, 19)
- ☐ (21, 18)
- ☐ (21, 17)
- ☐ (22, 16)
- ☐ (23, 16)
- ☐ (24, 17)

STOP

- ☐ (14, 11)
- ☐ (14, 18)
- ☐ (21, 6)
- ☐ (17, 6)

STOP

- ☐ (12, 1)
- ☐ (14, 3)
- ☐ (16, 3)
- ☐ (15, 2)
- ☐ (16, 1)

STOP

(X, Y)

- ☐ (4, 17)
- ☐ (5, 16)
- ☐ (6, 17)

STOP

- ☐ (8, 5)
- ☐ (9, 19)

STOP

- ☐ (20, 1)
- ☐ (22, 3)
- ☐ (24, 3)
- ☐ (23, 2)
- ☐ (24, 1)
- ☐ (25, 2)

STOP

- ☐ (7, 3)
- ☐ (3, 5)
- ☐ (23, 5)
- ☐ (22, 3)

STOP

(X, Y)

- ☐ (3, 18)
- ☐ (2, 17)
- ☐ (1, 18)

STOP

- ☐ (14, 5)
- ☐ (14, 6)

STOP

- ☐ (12, 1)
- ☐ (11, 2)
- ☐ (12, 3)
- ☐ (10, 3)
- ☐ (8, 1)
- ☐ (7, 2)
- ☐ (8, 3)
- ☐ (6, 3)
- ☐ (4, 1)

STOP

(X, Y)

- ☐ (9, 5)
- ☐ (9, 19)
- ☐ (17, 6)
- ☐ (9, 6)

STOP

- ☐ (2, 16)
- ☐ (3, 15)
- ☐ (4, 16)

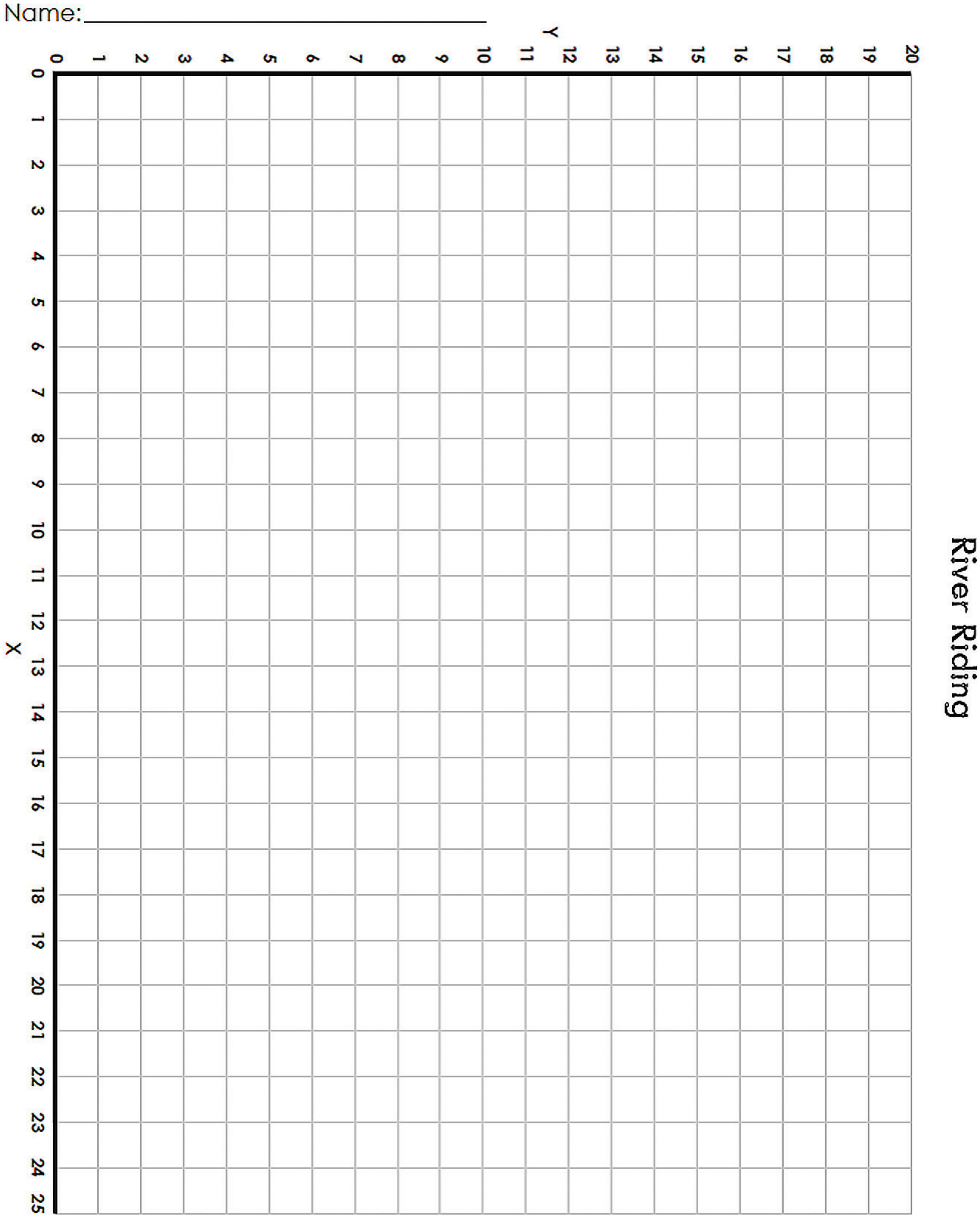
STOP

- ☐ (20, 1)
- ☐ (19, 2)
- ☐ (20, 3)
- ☐ (18, 3)
- ☐ (16, 1)

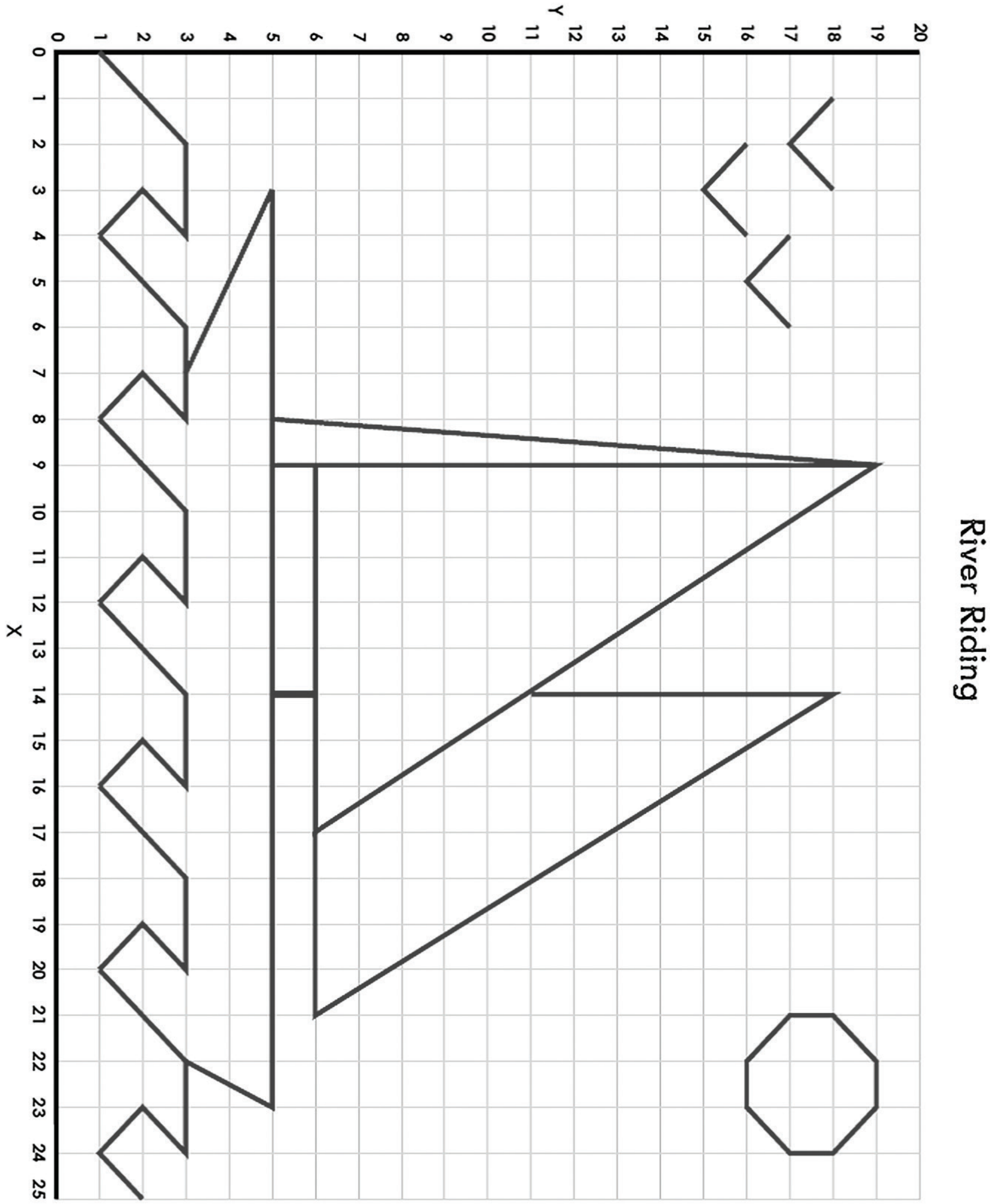
STOP

Now color your picture

Name:________________________________


River Riding
X
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
Y
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

What the picture should look like:



Name: ____________________________

A. I.

NOTE: In each section, do NOT connect the last point back to first point.

(X, Y)

- ☐ (1, -20)
- ☐ (1, -19)
- ☐ (3, -18)
- ☐ (5, -18)
- ☐ (7, -19)
- ☐ (7, -20)
- ☐ (1, -20)

STOP

- ☐ (-1, -20)
- ☐ (-1, -19)
- ☐ (-3, -18)
- ☐ (-5, -18)
- ☐ (-7, -19)
- ☐ (-7, -20)
- ☐ (-1, -20)

STOP

- ☐ (3, -14)
- ☐ (5, -14)
- ☐ (6, -13)
- ☐ (6, -12)
- ☐ (5, -11)
- ☐ (3, -11)
- ☐ (2, -12)
- ☐ (2, -13)
- ☐ (3, -14)

STOP

- ☐ (-3, -14)
- ☐ (-5, -14)
- ☐ (-6, -13)
- ☐ (-6, -12)
- ☐ (-5, -11)
- ☐ (-3, -11)
- ☐ (-2, -12)
- ☐ (-2, -13)
- ☐ (-3, -14)

STOP

(X, Y)

- ☐ (2, -5)
- ☐ (5, -2)
- ☐ (6, -2)
- ☐ (6, -8)
- ☐ (5, -9)
- ☐ (3, -9)
- ☐ (2, -8)
- ☐ (2, -5)

STOP

- ☐ (-2, -5)
- ☐ (-5, -2)
- ☐ (-6, -2)
- ☐ (-6, -8)
- ☐ (-5, -9)
- ☐ (-3, -9)
- ☐ (-2, -8)
- ☐ (-2, -5)

STOP

- ☐ (1, -4)
- ☐ (4, -1)
- ☐ (4, 1)
- ☐ (-4, 1)
- ☐ (-4, -1)
- ☐ (-1, -4)
- ☐ (1, -4)

STOP

- ☐ (-3, 4)
- ☐ (-5, 8)
- ☐ (-5, 12)
- ☐ (5, 12)
- ☐ (5, 8)
- ☐ (3, 4)
- ☐ (-3, 4)

STOP

(X, Y)

- ☐ (3, 19)
- ☐ (-3, 19)
- ☐ (-4, 18)
- ☐ (-4, 14)
- ☐ (-3, 13)
- ☐ (3, 13)
- ☐ (4, 14)
- ☐ (4, 18)
- ☐ (3, 19)

STOP

- ☐ (-6, 11)
- ☐ (-7, 12)
- ☐ (-9, 12)
- ☐ (-10, 11)
- ☐ (-10, 9)
- ☐ (-9, 8)
- ☐ (-7, 8)
- ☐ (-6, 9)
- ☐ (-6, 11)

STOP

- ☐ (6, 11)
- ☐ (7, 12)
- ☐ (9, 12)
- ☐ (10, 11)
- ☐ (10, 9)
- ☐ (9, 8)
- ☐ (7, 8)
- ☐ (6, 9)
- ☐ (6, 11)

STOP

- ☐ (-2, 19)
- ☐ (-2, 20)
- ☐ (2, 20)
- ☐ (2, 19)

STOP

- ☐ (2, 4)
- ☐ (2, 1)

STOP

(X, Y)

- ☐ (-6, 6)
- ☐ (-7, 7)
- ☐ (-9, 7)
- ☐ (-10, 6)
- ☐ (-10, 5)
- ☐ (-9, 4)
- ☐ (-7, 4)
- ☐ (-6, 5)
- ☐ (-6, 6)

STOP

- ☐ (6, 6)
- ☐ (7, 7)
- ☐ (9, 7)
- ☐ (10, 6)
- ☐ (10, 5)
- ☐ (9, 4)
- ☐ (7, 4)
- ☐ (6, 5)
- ☐ (6, 6)

STOP

- ☐ (6, 1)
- ☐ (6, -1)
- ☐ (7, -1)
- ☐ (7, 0)
- ☐ (8, 1)
- ☐ (9, 0)
- ☐ (9, -2)
- ☐ (10, -2)
- ☐ (10, 1)
- ☐ (9, 2)
- ☐ (7, 2)
- ☐ (6, 1)

STOP

- ☐ (2, 15)
- ☐ (2, 14)
- ☐ (-2, 14)
- ☐ (-2, 15)
- ☐ (2, 15)

STOP

(X, Y)

- ☐ (-6, 1)
- ☐ (-6, -1)
- ☐ (-7, -1)
- ☐ (-7, 0)
- ☐ (-8, 1)
- ☐ (-9, 0)
- ☐ (-9, -2)
- ☐ (-10, -2)
- ☐ (-10, 1)
- ☐ (-9, 2)
- ☐ (-7, 2)
- ☐ (-6, 1)

STOP

- ☐ (1, 16)
- ☐ (3, 16)
- ☐ (3, 18)
- ☐ (1, 18)
- ☐ (1, 16)

STOP

- ☐ (-1, 16)
- ☐ (-3, 16)
- ☐ (-3, 18)
- ☐ (-1, 18)
- ☐ (-1, 16)

STOP

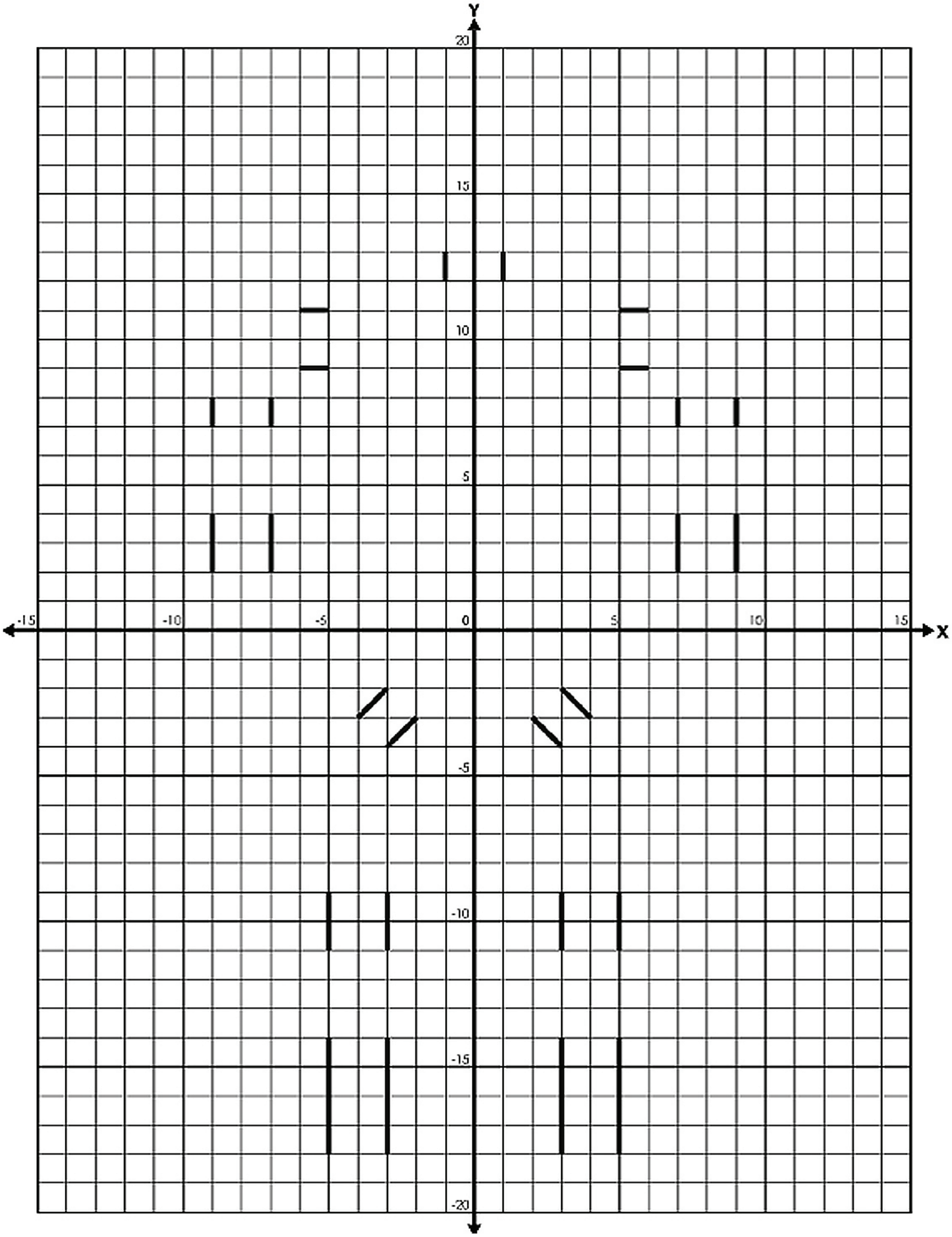
- ☐ (-3, 11)
- ☐ (-3, 8)
- ☐ (3, 8)
- ☐ (3, 11)
- ☐ (-3, 11)

STOP

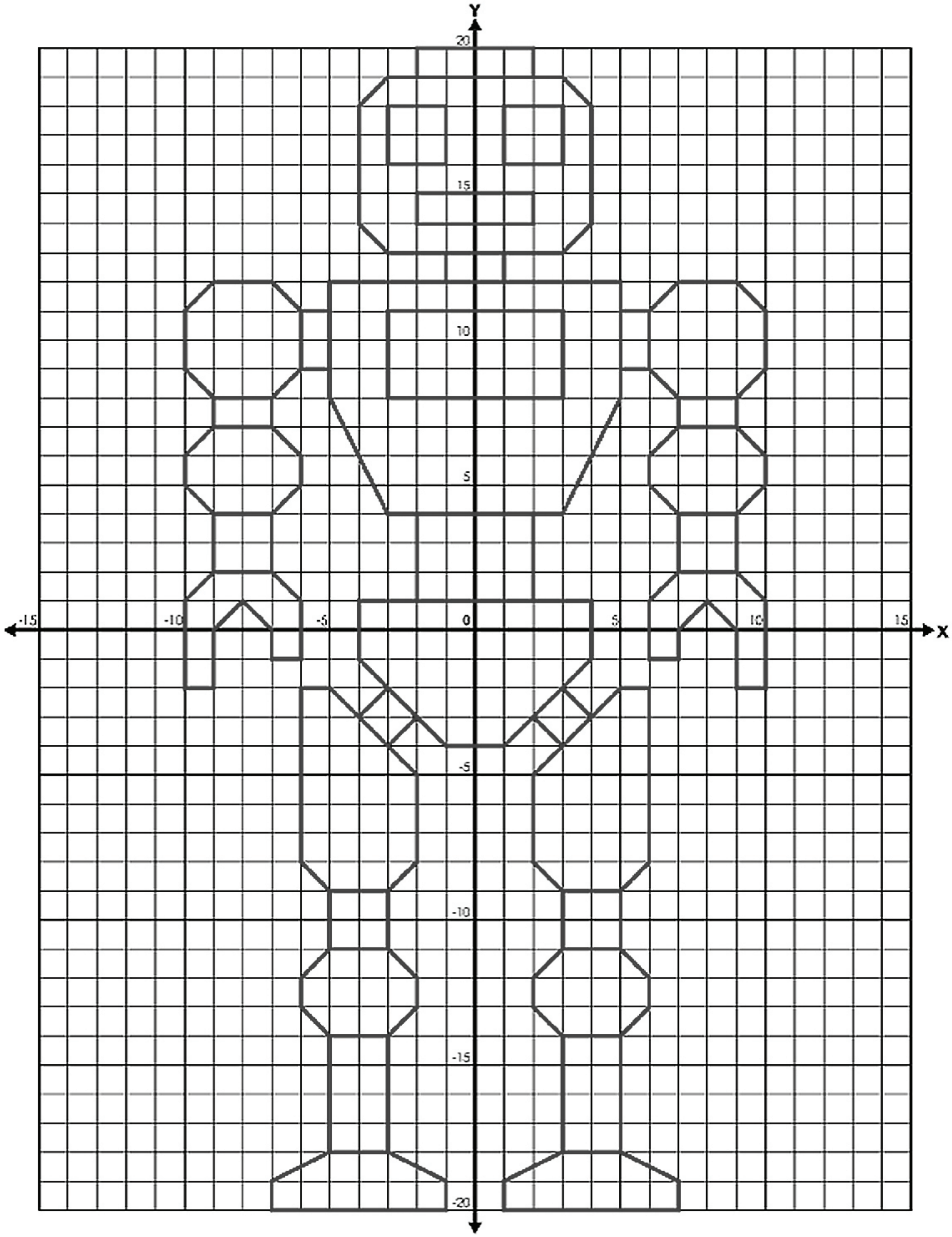
- ☐ (-2, 4)
- ☐ (-2, 1)

STOP

Now color your picture.


Y
20
15
10
5
0
-5
-10
-15
-20
X

What the picture should look like:



TOPIC 4: TRANSFORMATION GEOMETRY

INTRODUCTION

- This unit runs for 6 hours.
- It is part of the content area, 'Space and Shape' and counts for 25% in the final exam.
- The unit covers transformations of points and shapes on the Cartesian plane and creates an opportunity to practice this skill more.
- Learners must be encouraged to describe any transformation in two parts – the type of transformation as well as the more specific explanation.
  For example: A reflection about the y-axis.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 7	GRADE 8	GRADE 9/FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Recognise, describe and perform translations, reflections and rotations with geometric figures and shapes on squared paper • Identify and draw lines of symmetry in geometric figures • Draw enlargements and reductions of geometric figures on squared paper and compare them in terms of shape and size	• Recognise, describe and perform transformations with points on a co-ordinate plane, focusing on reflections of a point about the x and y-axes and translating a point within and across quadrants • Recognise, describe and perform transformations with triangles on a co-ordinate plane, focusing on the co-ordinates of the vertices when reflecting a triangle in the axes, translating a triangle across the quadrants and rotating a triangle about the origin • Use proportion to describe the effect of enlargement and reduction on area and perimeter of geometric figures	• Recognise, describe and perform transformations with points on a co-ordinate plane, focusing on reflections of a point about the x and y-axes; translating a point within and across quadrants; and reflection in the line $y = x$ • Use proportion to describe the effect of enlargement and reduction on area and perimeter of geometric figures • Investigate the co-ordinates of the vertices of figures that have been enlarged or reduced by a given scale factor

GLOSSARY OF TERMS

Term	Explanation / Diagram
Translation	A horizontal and/or vertical slide from one position to another. Every point moves the same distance and in the same direction. The translated shape is congruent to the original shape.
Reflection	A mirror image of a shape. In transformation geometry, reflections are done across the x-axis or the y-axis in the Cartesian Plane. Every point is the same distance from a central line. The reflected shape is congruent to the original shape.
Rotation	A turning of a shape around a certain point. In this section it is always around (from) the origin on a Cartesian Plane. Every point makes an imaginary circle from the point of rotation. The rotated shape is congruent to the original shape.
Congruent	Exactly the same size. All sides and all angles are equal.
Similar	Same shape but different size. Sides change length but all angles remain equal.
Enlargement	The resizing of a shape to make it bigger. The shape of the transformed shape will be similar to the original shape.
Reduction	The resizing of a shape to make it smaller. The shape of the transformed shape will be similar to the original shape.

SUMMARY OF KEY CONCEPTS

Before starting this section, keep in mind how important it is that learners understand how to plot points in a Cartesian plane. As this was introduced to them in the previous topic, it should be fresh in their minds but may also need to be revised.

Transformation means to change. Change could mean change in position or change in size of the shape. Both these types of transformation will be dealt with this year.

In primary school the words slide, flip and turn may have been used to describe the change in position of a shape. In Grade 8, the following transformations of co-ordinates and shapes will be dealt with:
Translation (slide)
Reflection (flip)
Rotation (turn)

Enlargements (which transform the size of a shape rather than a movement) will also be covered.

Translations

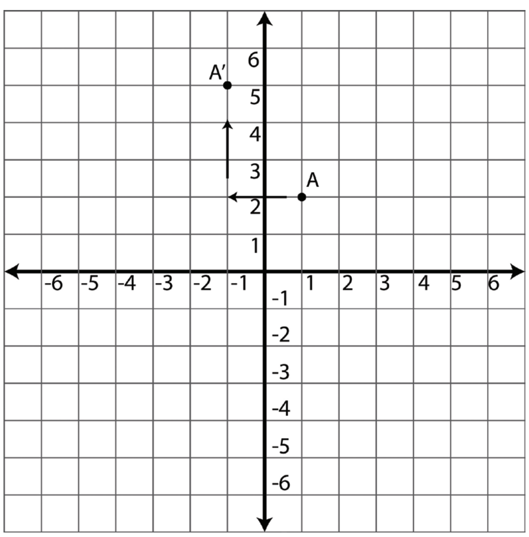
A translation is a horizontal and/or vertical slide from one point to another. The object or shape being translated will remain the same size.
The translation of a point on the Cartesian plane:
A (1 ; 2) moves to the left two units and up three units.
The new co-ordinate is represented by A'

$\therefore$ A (1 ; 2) $\rightarrow$ A' (1 – 2 ; 2 + 3)

$\therefore$

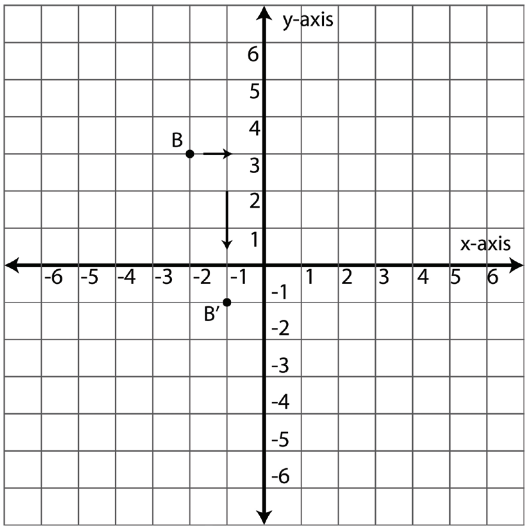
Horizontal shift Vertical shift

A' (-1 ; 5)



B (-2 ; 3) B' is the translation of B when moved one unit to the right and four units down.

$$B(-2;3) \rightarrow B'(-2+1;3-4)$$
$$\therefore (-1;-1)$$



If a point makes only a horizontal shift (left/right), only the x-co-ordinate will change.
If a point makes only a vertical shift (up/down), only the y-co-ordinate will change.

Examples:

P (2 ; -3) shifts 3 units to the left

$\therefore$ P' (2 – 3 ; -3 + 0)

$\therefore$ P' (-1 ; -3)

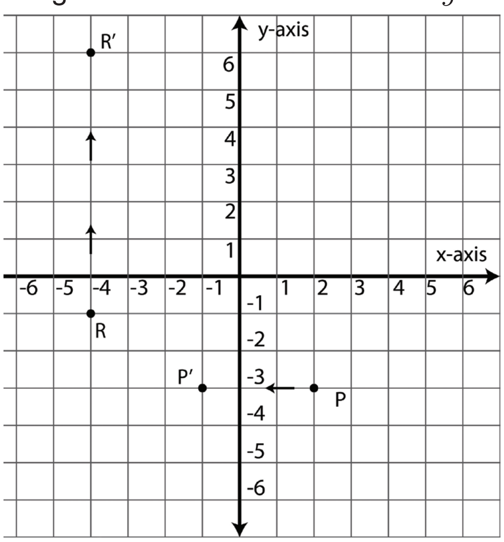
Note that the y-co-ordinate remained the same and only the x-co-ordinate changed

R (-4 ; -1) shifts 7 units up

$\therefore$ R' (-4 +0 ; -1 + 7)

$\therefore$ R' (-4 ; 6)

Note that the x -co-ordinate remained the same and only the y-co-ordinate changed



Reflections

A reflection is a mirror image. In Grade 8 we only deal with reflections about the x-axis and y-axis. This means that the axis involved will be like a mirror line.

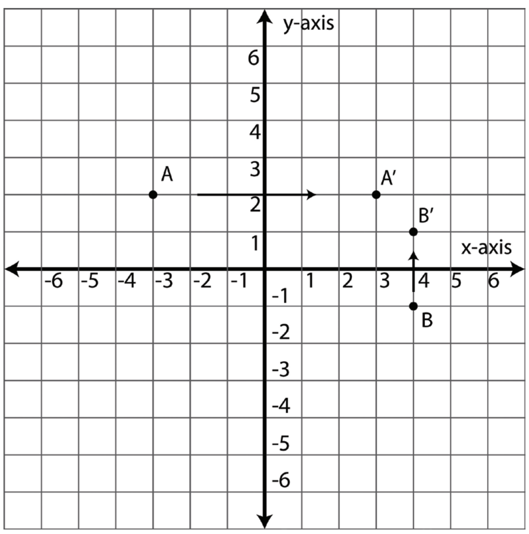
Examples of reflecting a co-ordinate:

(i) About the y-axis

A (-3 ; 2) → A' (3 ; 2) Note that a reflection in the y-axis will be a horizontal shift so only the x -co-ordinate will change

(ii) About the x-axis

B (4 ; -1) → B' (4 ; 1) Note that a reflection in the x-axis will be a vertical shift so only the y-co-ordinate will change



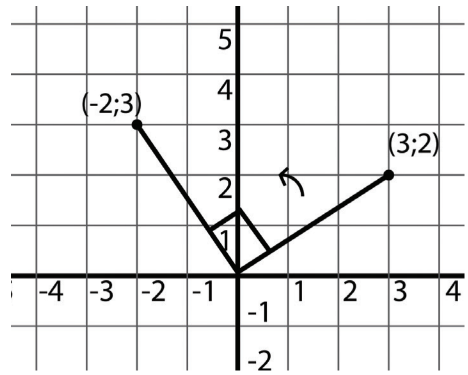
Teaching Tip: Ask learners to fold their page along the reflection line and check that the new point lies on top of the original point.

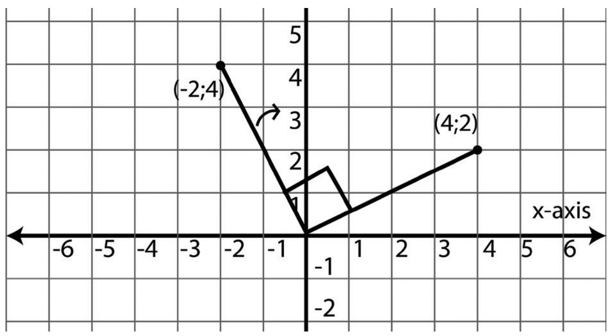
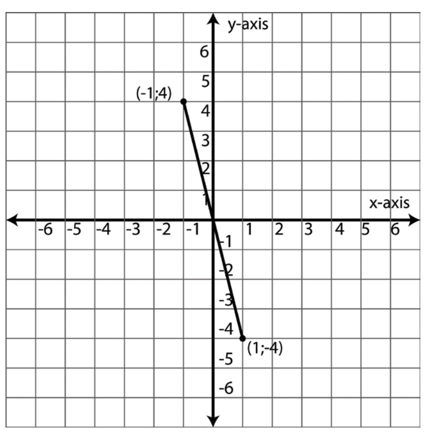
Rotations

1. In Grade 8, we will only deal with rotations about the origin. This means that the origin on the Cartesian plane will be the fixed point around which a shape will be rotated. In other words, if a compass was being used, the sharp point would remain at the origin.
2. You will be required to rotate a shape 90^{o} (clockwise and anti-clockwise) and 180^{o}
3. Before the rotating of points are discussed, please note that learners should be encouraged to establish the rules for themselves once they have experimented and observed what occurs to the co-ordinate when it has been rotated. Learners should NOT be taught the rules and asked to learn them.

(The explanation/rule on the right hand side of the following table are for the teacher's information only in order to create a better understanding for the teaching of this section)

<table>
<tr><td>90° anti-clockwise</td><td>The co-ordinates will swop over and the new x co-ordinate (which was the y co-ordinate) will change signs

$(x\,;y)$ $(-y\,;x)$
(3 : 2) (-2 : 3)</td></tr>
<tr><td colspan="2">
</td></tr>
<tr><td>90° clockwise</td><td>The co-ordinates will swop over and the new y co-ordinate (which was the x co-ordinate) will change signs

$(x\,;y)$ $(y\,;-x)$
(-2 : 4) (4 : 2)</td></tr>
<tr><td colspan="2">
</td></tr>
<tr><td>180°</td><td>The signs of each co-ordinate will change
$(x\,;y)$ $(-x\,;-y)$
(-1 : 4) (1 : - 4)</td></tr>
<tr><td colspan="2">
</td></tr>
</table>

Triangles – Translations, Rotations and Reflections

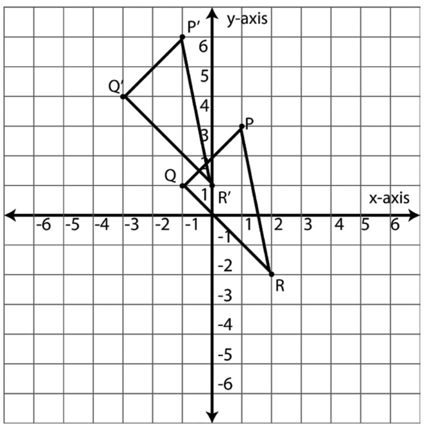
In order to translate a 2-dimensional shape, ALL points need to be moved according to the translation rule.

For example:
∆PQR is made up of the following points: P (1 ; 3) ; Q (-1 ; 1) and R (2 ; -2)

Draw ∆PQR on a Cartesian plane as well as ∆P'Q'R' on the same Cartesian plane if ∆PQR is translated 2 units to the left and 3 units up.

P' (1 – 2 ; 3 + 3) Q' (-1 -2 ; 1 + 3) R' (2 – 2 ; -2 + 3)

P' (-1 ; 6) Q' (-3 ; 4) R' (0 ; 1)

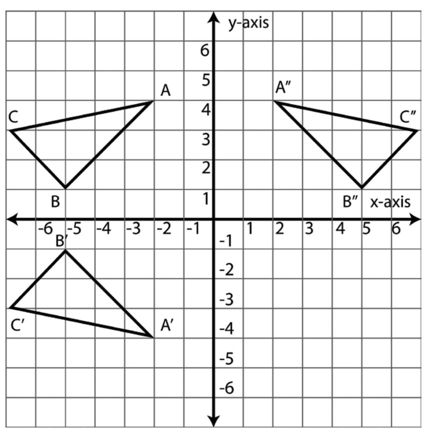


Teaching Tip: Tell learners to always check that their new shape is exactly the same size as the original shape.

In order to reflect a 2-dimensional shape, ALL points need to be moved according to the reflection rule.

For example:
∆ABC is made up of the points A (-2 ; 4) ; B (-5 ; 1) and C (-6 ; 3)

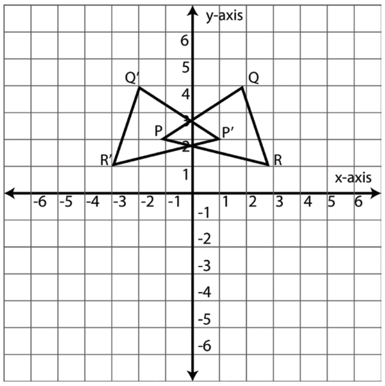
1. Draw ∆ABC on a Cartesian plane.
2. Draw ∆A'B'C', the reflection of ∆ABC in the x-axis.
3. Draw ∆A''B''C'' the reflection of ∆ABC in the y-axis.



Note: If a shape crosses over the axis that it is being reflected in, in its original form then when it has been reflected parts of it will overlap the original shape.

For example:
$\triangle$PQR is reflected in the y-axis:

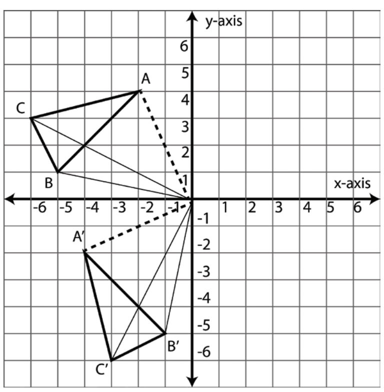
P (-1 ; 2)   Q (2 ; 4)   R (3 ; 1)
P' (1 ; 2)   Q' (-2 ; 4)   R' (-3 ; 1)



Teaching Tip: Tell learners to always check that their new shape is exactly the same size as the original shape.

In order to rotate a 2-dimensional shape, ALL points need to be moved according to the rotation rule.

For example:
$\triangle$ABC is made up of the points A (-2 ; 4) ; B (-5 ; 1) and C (-6 ; 3)
Draw $\triangle$ABC on a Cartesian plane.
Draw $\triangle$A'B'C', a rotation of $\triangle$ABC by 90° anti-clockwise through the origin
Rotated points: A' (-4 ; -2) ; B' (-1 ; -5) and C' (-3 ; -6)

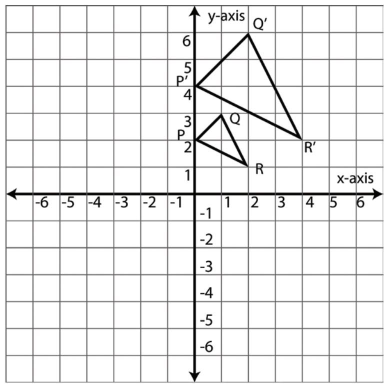


Notice the lines joining the corresponding points making a right angle.
Always ensure that your shape has remained exactly the same size!

Enlargements and Reductions

All enlargements will occur through/from the origin.
In other words, if you are required to enlarge a shape by a scale factor of 3, each co-ordinate will be multiplied by 3 to obtain the new co-ordinate.

For example:
If ΔPQR with, P (0 ; 2)   Q (1 ; 3)   R (2 ; 1) is enlarged by a scale factor of 2.
$\therefore$ $\Delta P'Q'R'$ P' (0 ; 4)   Q' (2 ; 6)   R' (4 ; 2)



TOPIC 5: GEOMETRY OF 3D OBJECTS

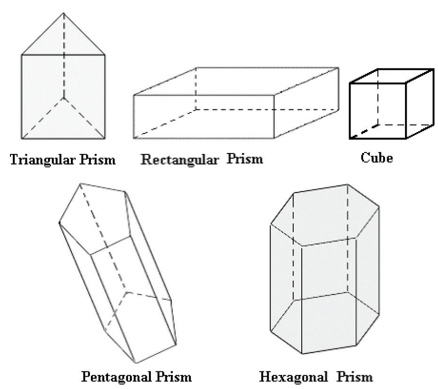
INTRODUCTION

- This unit runs for 7 hours.
- It is part of the content area, 'Space and Shape' and counts for 25% in the final exam.
- The unit covers the classification and building of 3D objects.
- The purpose of needing a good knowledge of 3D objects is simply that we live in a three-dimensional world. Every object you can see or touch has three dimensions that can be measured: length, width and height. The room you are sitting in can be described by these three dimensions.

SEQUENTIAL TEACHING TABLE

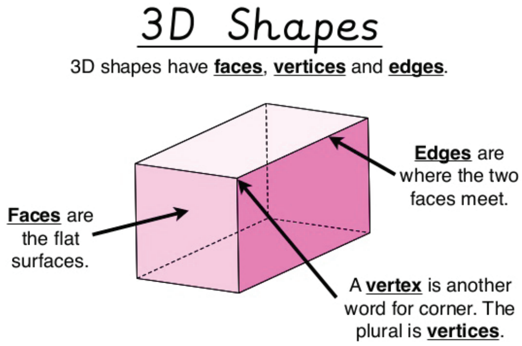
INTERMEDIATE PHASE/GRADE 7	GRADE 8	GRADE 9/FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Describe, sort and compare polyhedra in terms of shape and number of faces; number of vertices; number of edges • Use nets to create models of geometric solids, including cubes and prisms	• Describe, name and compare the 5 platonic solids in terms of shape and number of faces; number of vertices and number of edges • Use nets to create models of geometric solids, including cubes, prisms and pyramids	• Revise properties of the 5 platonic solids in terms of shape and number of faces; number of vertices and number of edges • Recognise and describe the properties of spheres and cylinders • Use nets to create models of geometric solids, including cubes, prisms, pyramids and cylinders

GLOSSARY OF TERMS

Term	Explanation / Diagram
2D	2-dimensional A shape that is made up of length and width (2 dimensions).
3D	3-dimensional A shape that is made up of length, width and height (3 dimensions).
Congruent	Exactly the same size.
Polygon	A 2D shape in which all the sides are made up of line segments. A polygon is given a name depending on the number of sides it has. Example: A 5 sided polygon is called a pentagon.
Solid	An object that occupies space (3-dimensional).
Platonic Solid	A solid shape where all the faces are congruent and all the edges are the same length. The cube is the most common platonic solid. Other platonic solids: tetrahedron (4 triangular faces); octahedron (8 triangular faces); icosahedron (20 triangular faces); dodecahedron (12 pentagonal faces).
Polyhedron	A solid in which all the surfaces (faces) are flat.
Prism	A solid with parallel equal bases. The bases are both polygons.
Right Prism	A prism which has the sides at right angles to the base. 
Face	A flat surface of a prism.
Edge	Where the faces of a prism meet.
Vertex	Where the edges of a prism meet (the corner).
Cube	A solid with six equal square faces.
Cuboid/Rectangular Prism	A solid with six rectangular faces.
Triangular prism	A solid with two equal triangular faces (one is the base) and three rectangular faces.
Cylinder	A solid with two equal circular faces (one is the base) and one rectangle (curved).
Net	A 2D shape, that when folded forms a 3D shape.
Pyramid	A solid object where the base is a polygon and the side faces are all triangles that meet at an apex (point). Pyramids are named according to their base, for example, square pyramid or triangular pyramid.
Apex	A point where the edges of a pyramid meet. The point at the top of a pyramid.

SUMMARY OF KEY CONCEPTS

Every 3-dimensional solid is made up of faces, edges and vertices.

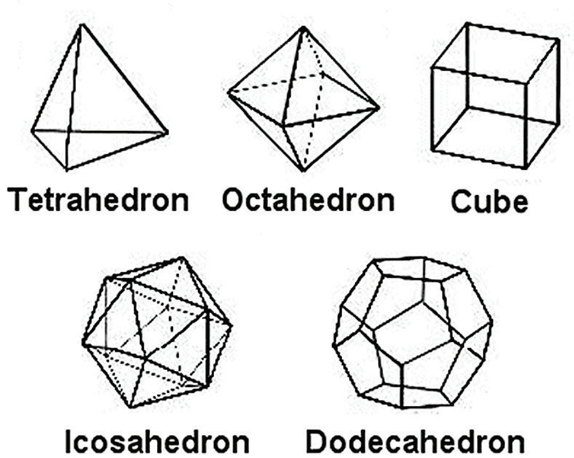


Teaching Tip: Use as many manipulatives as possible (ask learners to bring 3D objects from home) and spend some time allowing the learners to hold the objects and count the faces, edges and vertices.

Teaching Tip: Tell learners that they could draw a face on a face – this may help them remember where the faces are. (It isn't possible to draw on an edge or a vertex)

Platonic Solids

There are 5 platonic solids:



Note: the cube is also known as a hexahedron (taken from the word hexagon which means six-sided and the cube has 6 faces). The cube itself is a special hexahedron as all six faces are congruent. The square prism and rectangular prism are also classed as hexahedron.

Platonic Solids are special 3-dimensional shapes.

The following is true of all five of them:

- Each platonic solid is made up of congruent faces
- Each edge is equal in length
- Each vertex has the same number of edges meeting at it

Note: Learners must be able to count the faces, edges and vertices of all 5 of the platonic solids

Classifying 3D objects – Euler's law

Euler's law states the following:
For any polyhedron (a solid figure with many plane faces, typically more than six) the:

- number of faces
- plus the number of vertices (corners)
- minus the number of edges

always equals 2!

This can be written: **F + V − E = 2**

For example, consider the triangular prism. It has 5 faces, 6 vertices and 9 edges

$$5 + 6 - 9 = 2$$

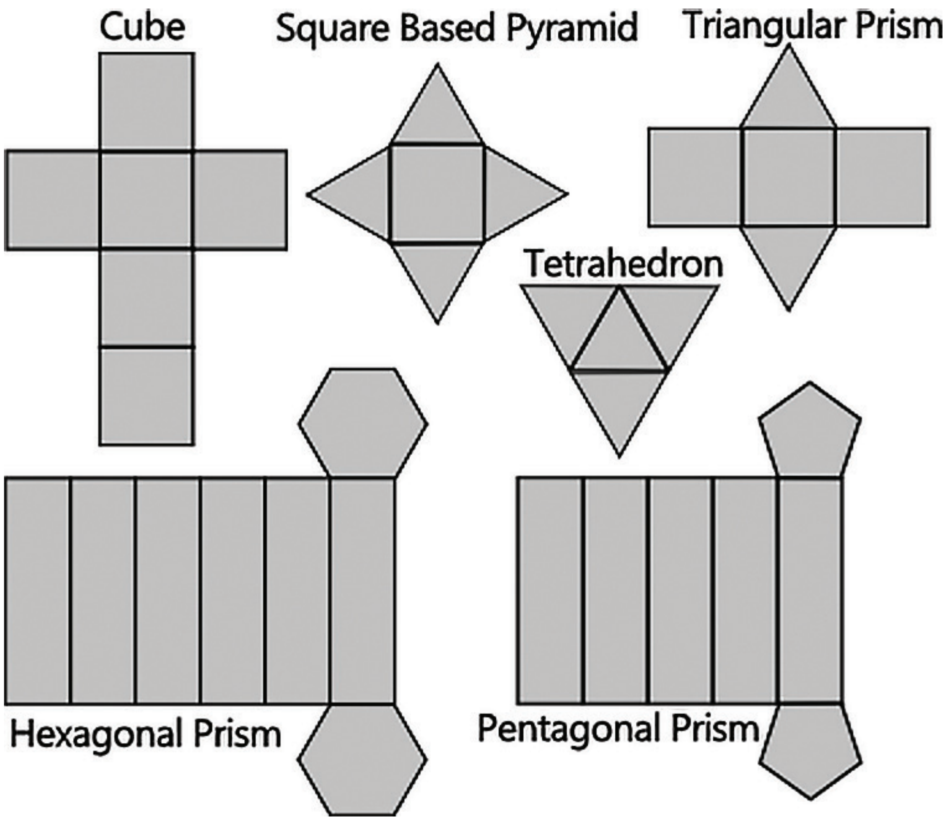
Using a table similar to the one in the resource section at the end of the topic, encourage learners to find Euler's law for themselves by completing the table and answering the questions that follow the table.

Nets and their 3D solids

In order for a 3-dimensional shape to be made, a 2-dimensional net is required.

A net is the shape that needs to be cut out in order to fold up and become a 3-dimensional shape.

In Term 3, surface area and volume of right prisms were covered so learners have already encountered nets and how they are related to 3D objects.
The following diagram shows the nets of some of the shapes dealt with in this topic:



Learners need to be able to match a net with its 3D shape as well as make a net for a 3D shape.

It is also important that learners draw the nets of 3D objects to scale which also links back to the constructions covered earlier in the year. For example, to draw the net of a triangular prism with an equilateral triangle as a base, learners will have to revise constructing an equilateral triangle.
Remember that Surface Area of a 3-dimensional shape is directly linked to the shape's 2-dimensional net.

Prisms and Pyramids

Prisms have:

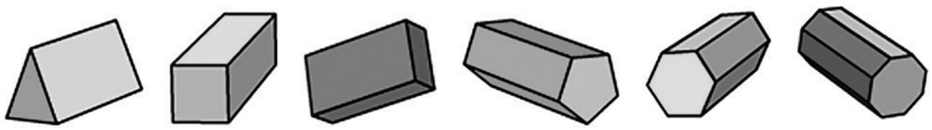
- Two congruent bases (which are parallel to each other)
- Rectangular side faces

Pyramids have:

- One base
- Triangular side faces that meet at one vertex

A prism has a polygonal base and a face opposite and congruent to the base, while a pyramid has a polygonal base and an apex (point) at the opposite side.

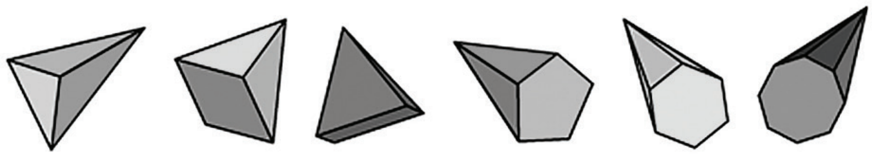
Examples of prisms



triangular prism | square prism | rectangular prism | pentagonal prism | hexagonal prism | octagonal prism

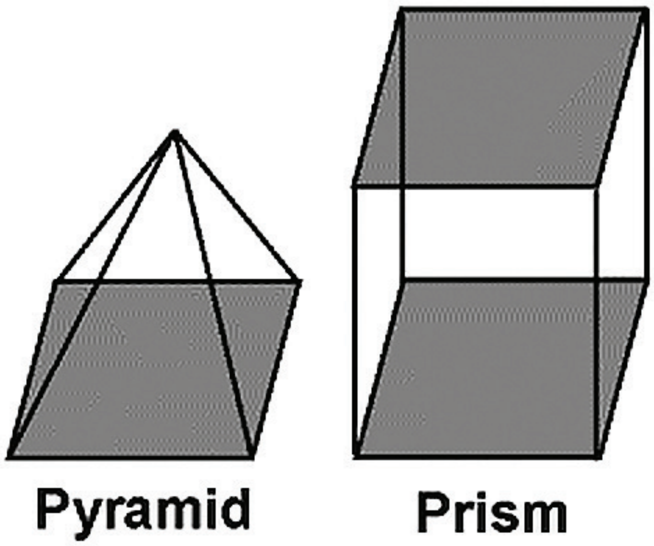
The edge of the pyramid's base and its apex form a triangle, and the number of sides of its base dictate how many triangular faces are present in the pyramid.

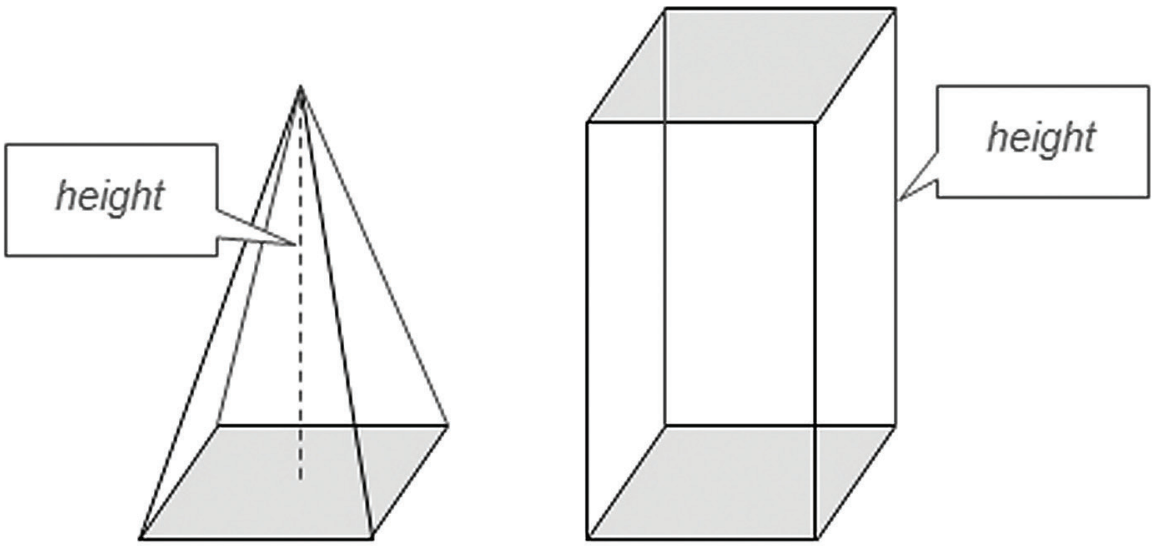
Examples of pyramids

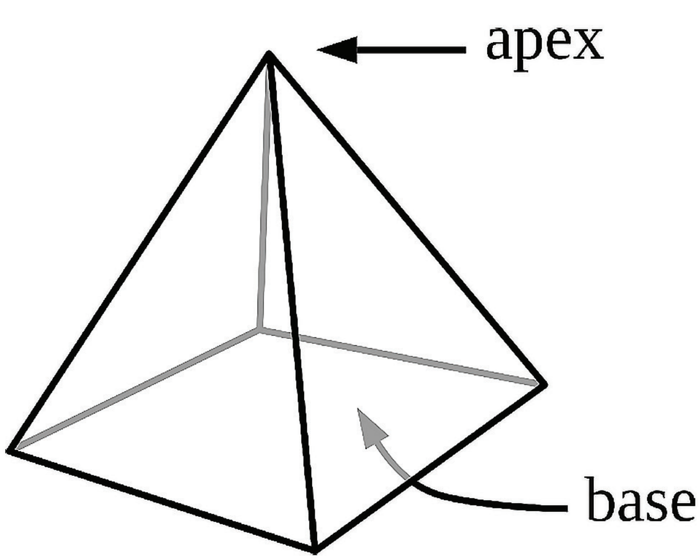


triangular pyramid | square pyramid | rectangular pyramid | pentagonal pyramid | hexagonal pyramid | octagonal pyramid

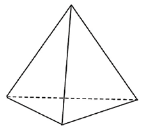
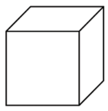
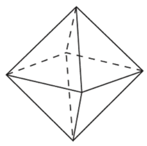
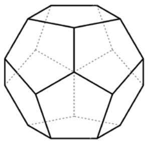
TOPIC 5 RESOURCES







Complete the following table then answer the questions:

Platonic solid	Number of Faces	Number of Vertices	Number of Edges
 Tetrahedron			
 Hexahedron (Cube)			
 Octahedron			
 Dodecahedron			
 Icosahedron			

1. Look at the first shape in the table above. Find the sum of the number of faces and the number of vertices. How does this total compare with the number of edges?
2. Now look at the other shapes in the table. Compare the sum of the number of faces and vertices with the number of edges. What did you find out? Is there a rule for all of the shapes?
3. So what is Euler's formula?

TOPIC 6: PROBABILITY

INTRODUCTION

- This unit runs for 4.5 hours.
- It is part of the content area, 'Data Handling' and counts for 10% in the final exam.
- The unit covers finding the probability of certain events. This is the chance of an event happening.
- It is important to note that having a good understanding of probability is one way to think about your world and the decisions you make every day. In living our lives, we often take on risks and expose ourselves to dangers (not necessarily physical but perhaps losing money for example). We may try things which we think will probably succeed, but we're not really sure. Probability theory gives us a way to think about these decisions and may help to take control of them.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 7	GRADE 8	GRADE 9/ FET PHASE
LOOKING BACK	**CURRENT**	**LOOKING FORWARD**
Perform simple experiments where the possible outcomes are equally likely and: • List the possible outcomes based on the conditions of the activity • Determine the probability of each possible outcome using the definition of probability	Consider a simple situation (with equally likely outcomes) that can be described using probability and: • List all the possible outcomes • Determine the probability of each possible outcome using the definition of probability • Predict with reasons the relative frequency of the possible outcomes for a series of trials based on probability • Compare relative frequency with probability and explain possible differences	Consider situations with equally probable outcomes and: • Determine probabilities for compound events using two-way tables and tree diagrams • Determine the probabilities for outcomes of events and predict their relative frequency in simple experiments • Compare relative frequency with probability and explain possible differences

GLOSSARY OF TERMS

Term	Explanation / Diagram
Probability	The likelihood of something happening. Usually expressed as a fraction (but could also be as a decimal or percentage) A probability answer is ALWAYS in the following range: $0 \leq x \leq 1$ (ie answer can only be zero, one or a common fraction).
Trial/Experiment	The process of trying something for a particular purpose. For example: Tossing a coin 100 times.
Outcome	A possible result from an experiment For example: 'tails' is one of two possible outcomes when tossing a coin.
Experimental probability	The result of doing an experiment to find the chances of an event occurring. For example: An experiment was conducted to see how many tails appeared when a coin was tossed 100 times. The result was . $\frac{47}{100}$
Relative Frequency	The outcome of an experiment. In the above example $\frac{47}{100}$ is the relative frequency.
Theoretical probability	The probability of an event happening using knowledge of numbers. For example: The chance of getting tails when tossing a coin is $\frac{1}{2}$ because 'tails' is the outcome being considered from a possible two outcomes.

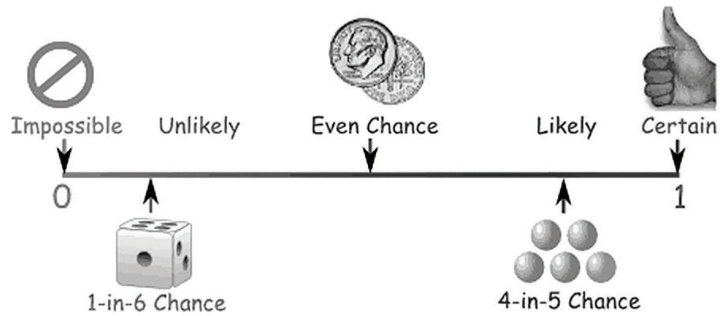
SUMMARY OF KEY CONCEPTS

An important concept of probability that needs to be understood from the beginning is that no answer to a probability question can ever be less than zero or bigger than one.

Every answer will always lie somewhere from zero to 1. Therefore, most answers are fractions except those that are actually zero or one.
Remember that fractions can be written in more than one way.

For example, $\frac{1}{2} = 0,5 = 50\%$

If there is an equally likely chance of an event happening (also known as a 50/50 chance), any one of the above fractions would represent the probability of such an event occurring. (Remember that 50% means 50 divided by 100 and is therefore not a number bigger than one although it may at first appear to be so)



Finding the probability of an event

To find the probability of an event occurring, the following calculation is used:

$$\frac{number\ of\ favourable\ outcomes}{total\ number\ of\ possible\ outcomes}$$

For example:
1. The probability of rolling a four when a die is rolled is $\frac{1}{6}$.

   Reason: a four is only <u>one</u> possible outcome from a total possible outcomes of <u>six.</u>
2. The probability of rolling an odd number when rolling a die is $\frac{3}{6} = \frac{1}{2}$

   Reason: A one, three or five (the odd numbers) represents <u>three</u> possible outcomes of a total of <u>six</u> outcomes.

Answers should always be simplified.

NOTE: Although this is preferable, remember that during the teaching of this topic it is best to focus more on the new skills being learnt and an understanding of where the numerator and denominator come from in a probability solution is more important than remembering to simplify a fraction.

Relative Frequency and probability

Ensure that learners understand the difference between relative frequency and probability.

Relative frequency could change every time you do an experiment. If different people were doing the same experiment, the relative frequency results could be different.

Probability however will be the same as it is theoretical.

For example:
If a coin is tossed 20 times by two different people, it is likely that they will not get the same results. Person A could get 9 tails and 11 heads while Person B could get 8 tails and 12 heads.
Person A would say that there is a $\frac{9}{20}$ chance of getting a tail while Person B would argue that there is a $\frac{8}{20}$ $\left(\frac{2}{5}\right)$ chance of getting a tail.
Theoretically, the probability of getting a tail is $\frac{1}{2}$ as there should be a 50% chance of getting one of the two possible outcomes.
The more times you repeat the activity involved in the experiment the closer the relative frequency should be to the probability. (Tossing a coin 500 times is likely – note the probability word here - to produce a result of getting tails very close to $\frac{1}{2}$ whereas tossing the coin only 10 times could produce a result of $\frac{7}{10}$)

Notes

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